Cavity Control in a Single-Electron Quantum Cyclotron

An Improved Measurement of the Electron Magnetic Moment

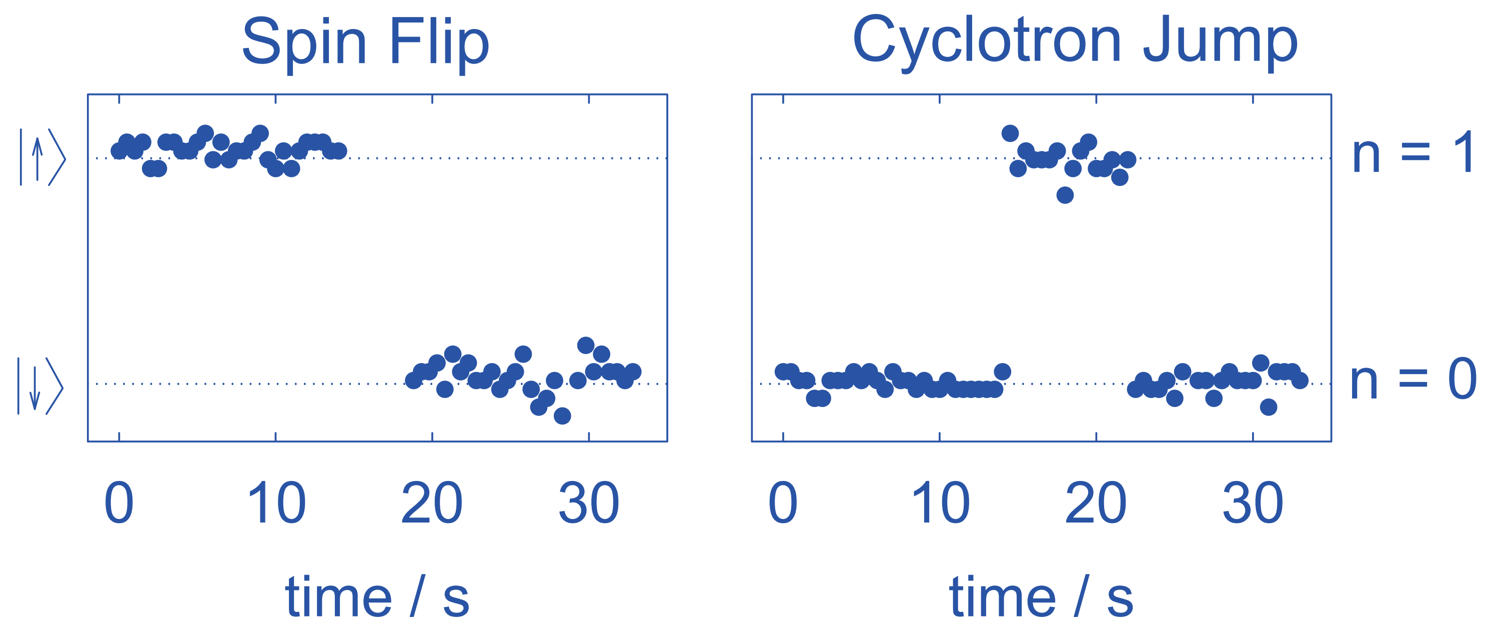
David Hanneke

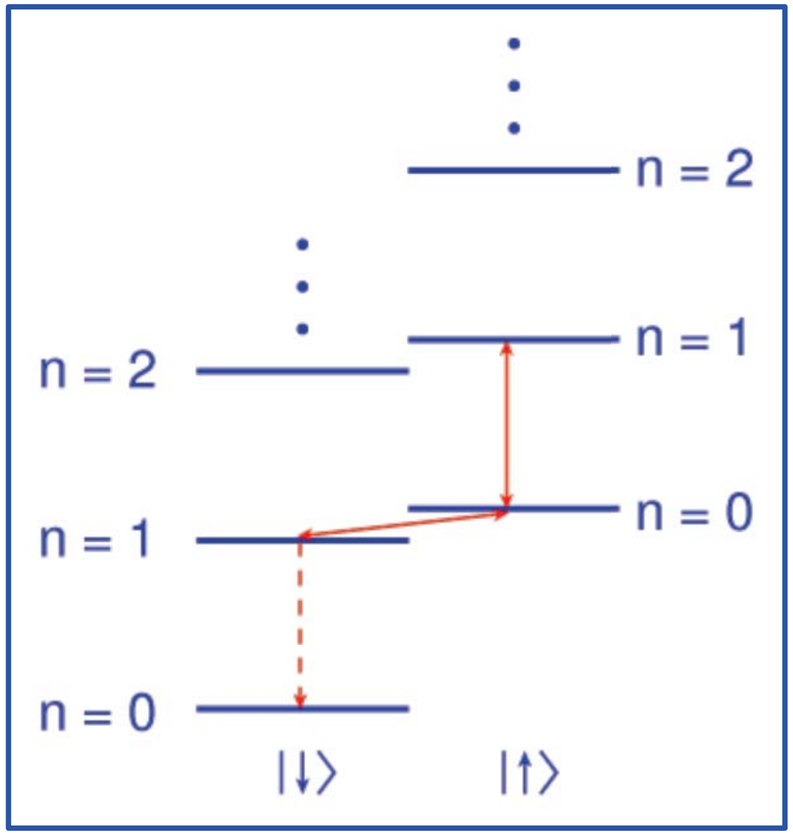
Michelson Postdoctoral Prize Lectures

13 May 2010

The Quantum Cyclotron

- Single electron
- Resolve lowest cyclotron and spin states via QND measurement





Acknowledgements

<u>Principal Investigator</u>

Gerald Gabrielse

<u>Postdocs</u>

Maarten Jansen
Kamal Abdullah

20 years
7 theses

<u>Graduate Students</u>

Josh Dorr
Shannon Fogwell
David Hanneke (2007)
Brian Odom (2004)
Brian D’Urso (2003)
Steve Peil (1999)
Daphna Enzer (1996)
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Joseph Tan (1992)

Outline

I. Introduction
II. Measurement overview
III. Novel techniques
IV. Measurement details
V. Uncertainties
VI. Experimental challenges
VII. What's next

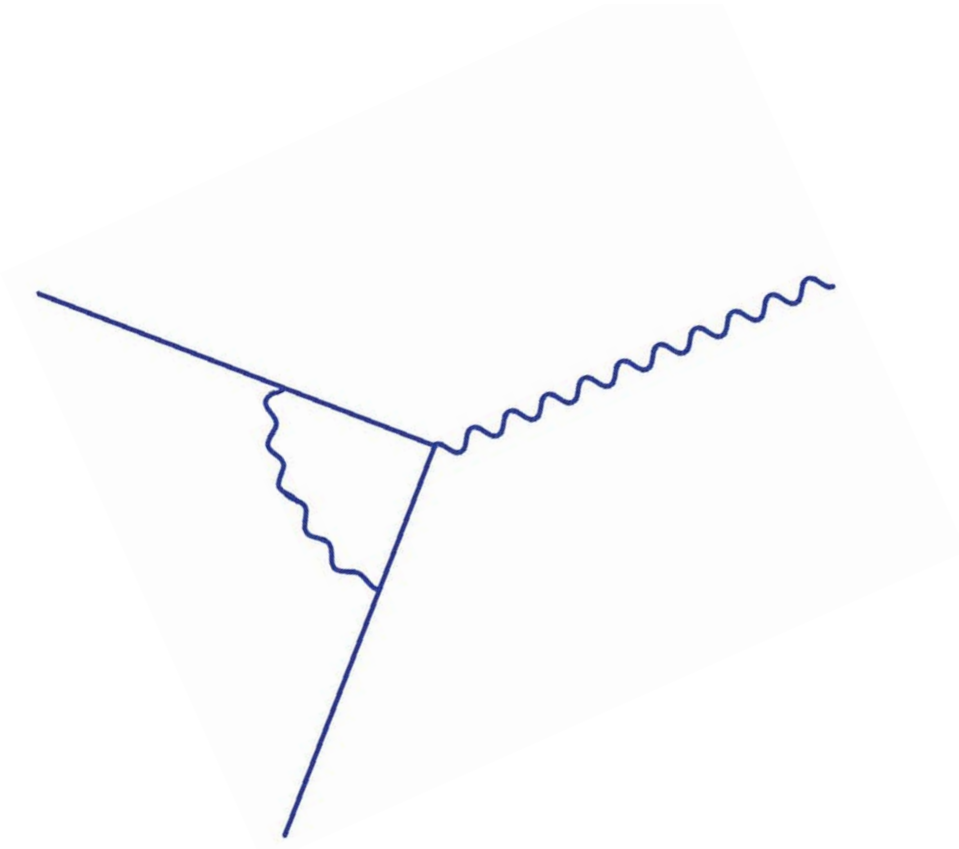
Magnetic Moments

$$\vec{\mu} = g \frac{-e\hbar}{2m} \frac{\vec{S}}{\hbar}$$

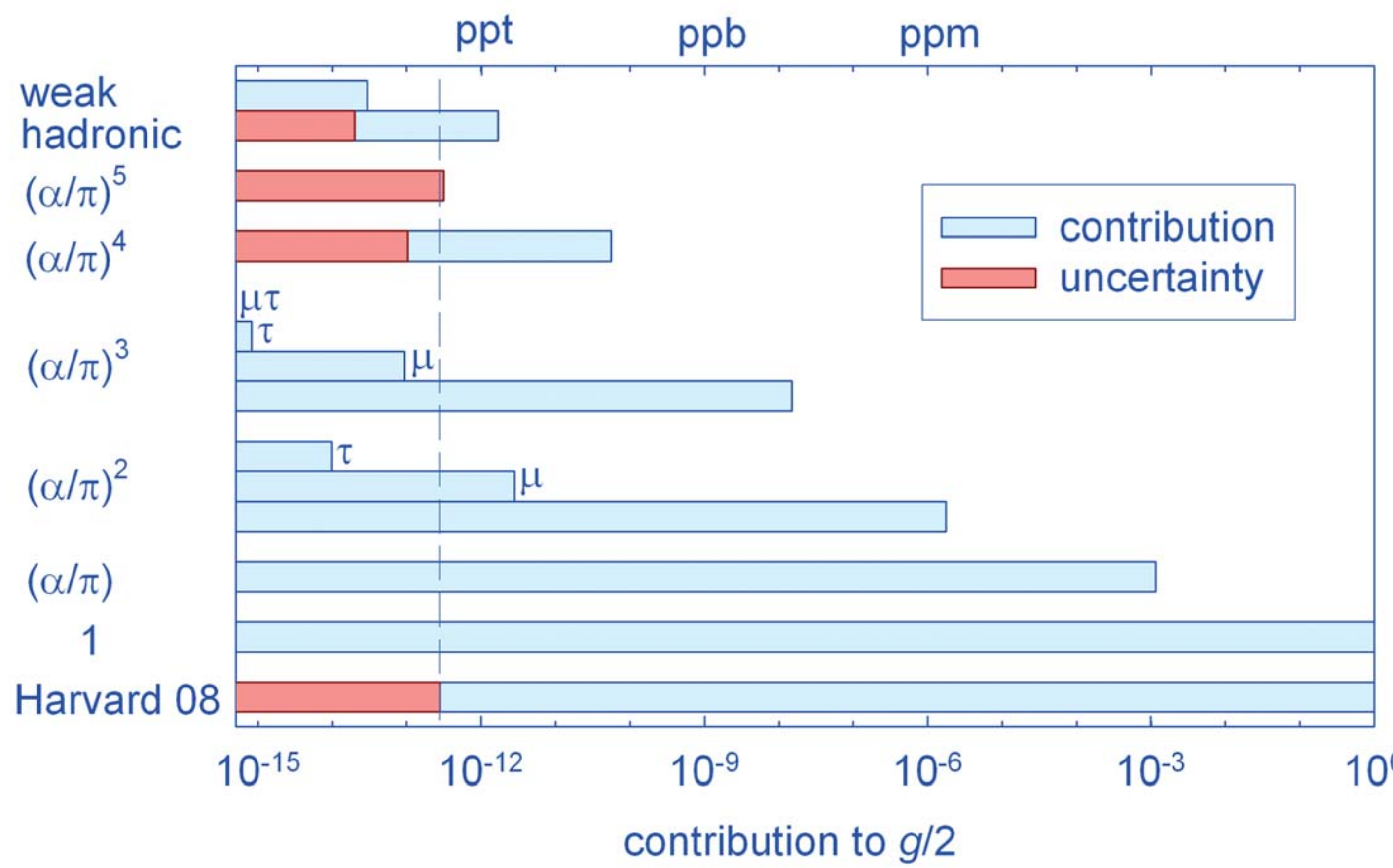
	Orbital angular momentum	Intrinsic angular momentum (Dirac point particle)	Structure of the vacuum (QED)	Structure of the particle (proton)
g =	1	2	2.002 319 304 …	5.585 …

α, *g*, and QED

$$\frac{g}{2} = 1 + \underbrace{C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + \ldots + a^{\mu,\tau} + a^{had} + a^{weak}}_{a \equiv \frac{g-2}{2} \approx 10^{-3}}$$



$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$$



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Exact Values for C_i

$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + \ldots + a^{\mu,\tau} + a^{had} + a^{weak}$$

$$C_2 = \frac{1}{2} = 0.5$$

1 Feynman diagram

$$C_4 = \frac{197}{144} + \frac{\pi^2}{12} + \frac{3}{4}\zeta(3) - \frac{1}{2}\pi^2 \ln 2 = -0.328\ \ldots$$

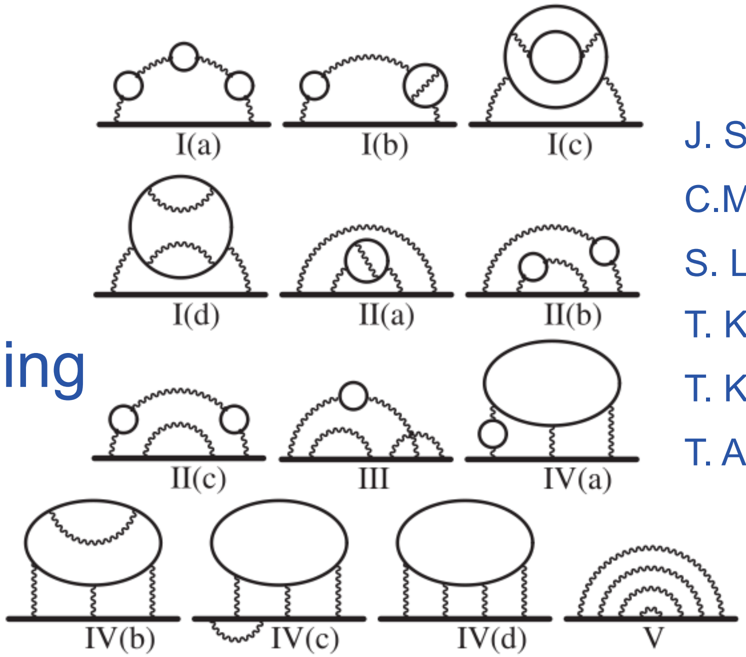
7 diagrams

$$C_6 = \frac{83}{72}\pi^2\zeta(3) - \frac{215}{24}\zeta(5) + \frac{100}{3}\left[\left(\sum_{n=1}^{\infty}\frac{1}{2^n n^4} + \frac{1}{24}\ln^4 2\right) - \frac{1}{24}\pi^2\ln^2 2\right] - \frac{239}{2160}\pi^4 + \frac{139}{18}\zeta(3) - \frac{298}{9}\pi^2\ln 2 + \frac{17101}{810}\pi^2 + \frac{28259}{5184} = 1.181\ \ldots$$

72 diagrams

C_8 has 891 diagrams
- Known to 0.2 % numerically
- Exact calculation underway

C_{10} has 12 672 diagrams
- Numeric calculation just beginning
- Largest uncertainty in α !

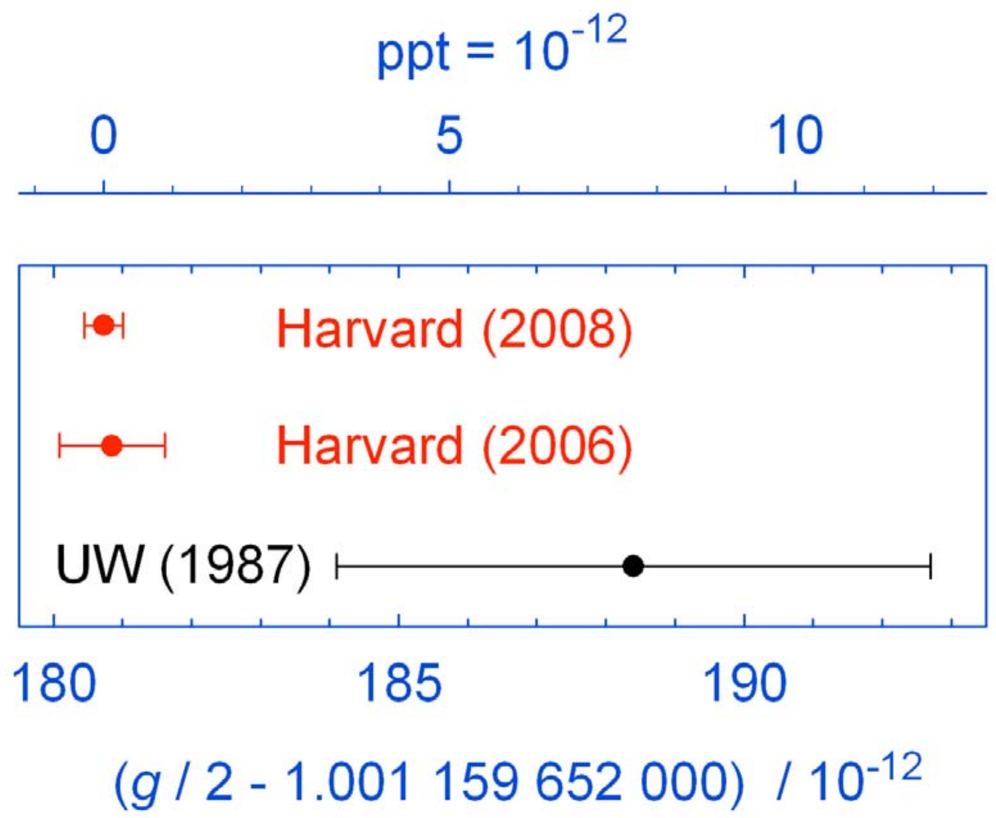


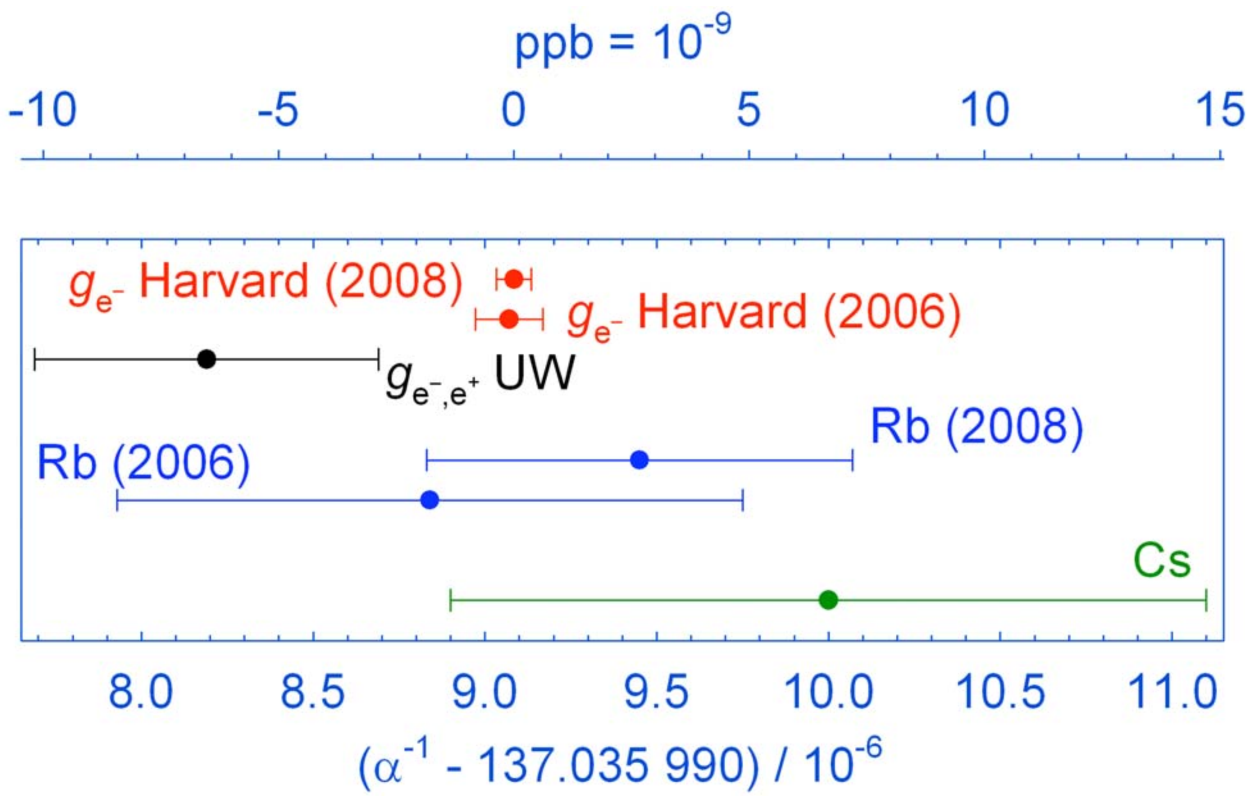
J. Schwinger, *Phys. Rev.* **73**, 416 (1948)
C.M. Sommerfield, *Phys. Rev.* **107**, 328 (1957)
S. Laporta and E. Remiddi, *Phys. Lett. B* **379**, 283 (1996)
T. Kinoshita and M. Nio, *Phys. Rev. D* **73**, 013003 (2006)
T. Kinoshita and M. Nio, *Phys. Rev. D* **73**, 053007 (2006)
T. Aoyama *et al*, *Phys. Rev. Lett.* **99** 110406 (2007)

Measurement Results

$g/2$ = 1.001 159 652 180 73 (28) [0.28 ppt]

α^{-1} = 137.035 999 084 (51) [0.37 ppb]





D. Hanneke, S. Fogwell, and G. Gabrielse, *Phys. Rev. Lett.* **100**, 120801 (2008)

B. Odom, D. Hanneke, B. D'Urso, and G. Gabrielse, *Phys. Rev. Lett.* **97**, 030801 (2006)

G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, and B. Odom, *Phys. Rev. Lett.* **97**, 030802 (2006). *Ibid*. **99** 039902(E) (2007)

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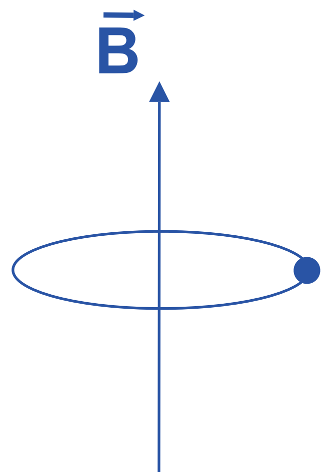
Experimenter's *g*

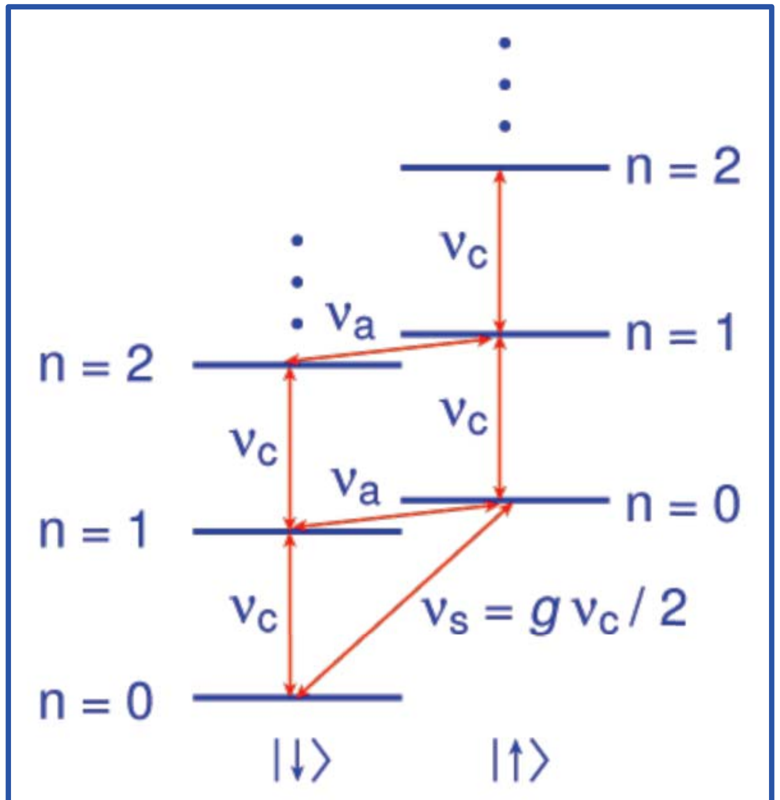
g in free-space (with a magnetic field)

What should we measure? Frequency!



$$\nu_c = \frac{eB}{2\pi m}$$

$$\nu_s = \frac{g}{2}\nu_c$$

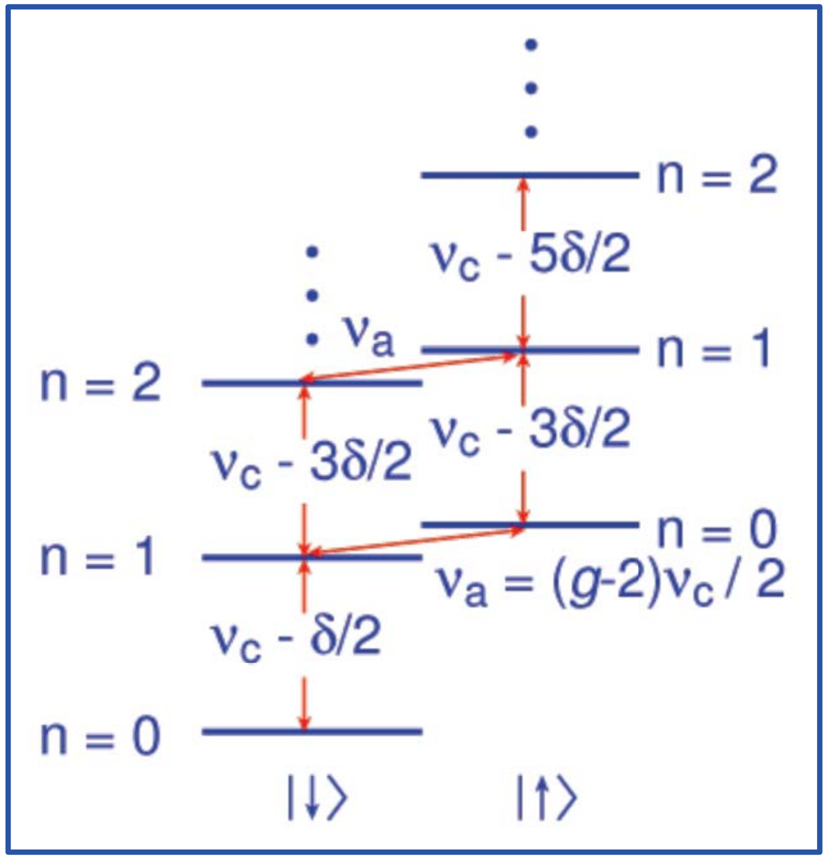


$$\frac{g}{2} = \frac{\nu_s}{\nu_c} = 1 + \frac{\nu_s - \nu_c}{\nu_c} = 1 + \frac{\nu_a}{\nu_c}$$

Experimenter's *g*

Special Relativistic corrections

$$\frac{\delta}{\nu_c} = \frac{h\nu_c}{mc^2} \approx 10^{-9}$$

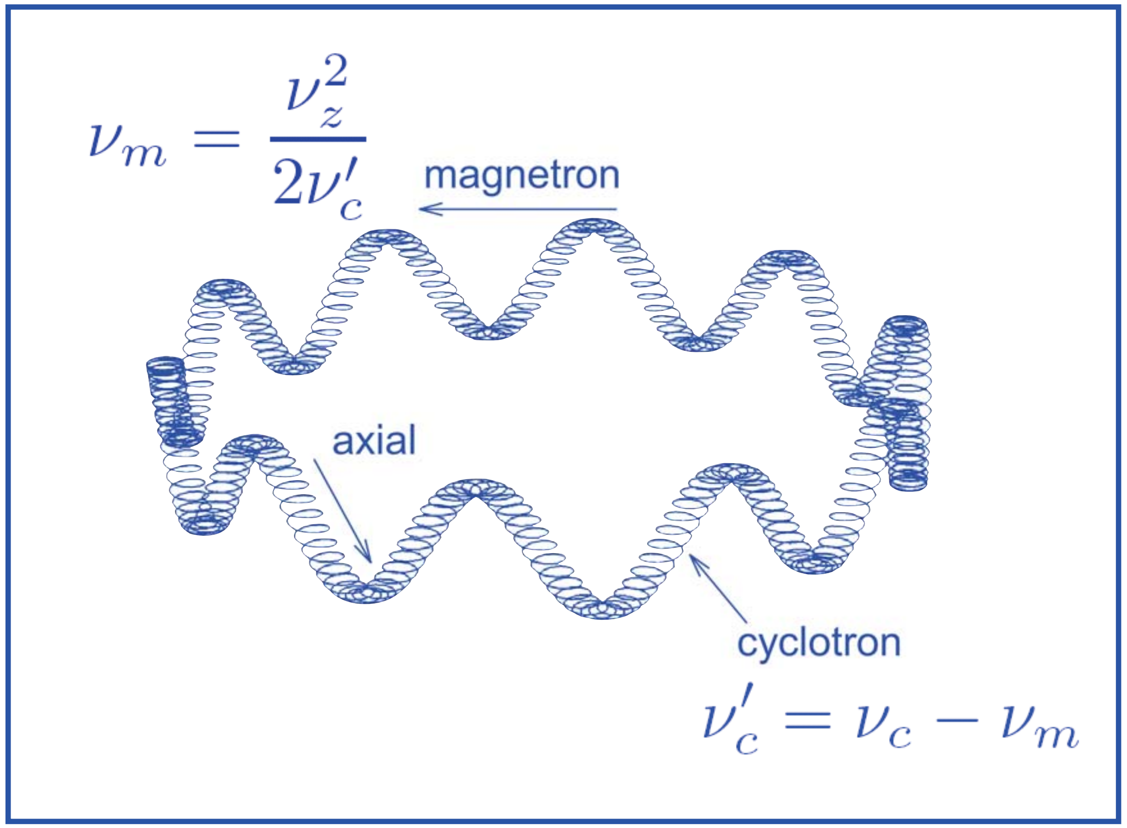


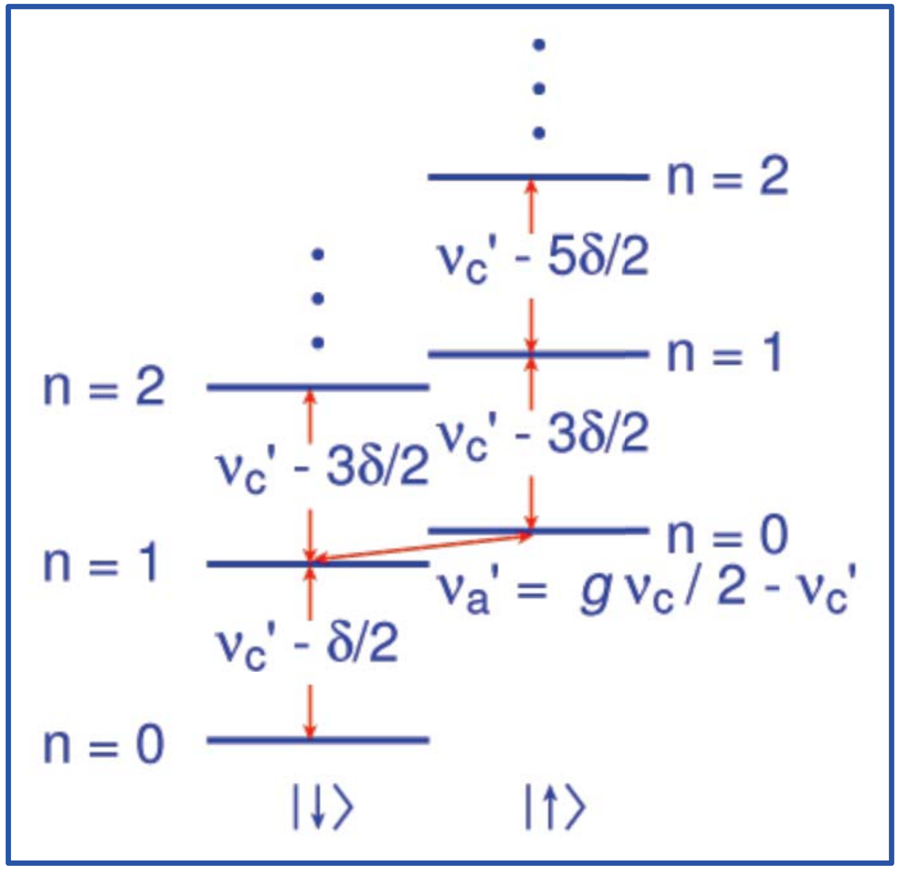
- The cyclotron is an anharmonic oscillator.
- No relativistic correction to ν_a.

Experimenter's *g*

Penning Trap's electrostatic quadrupole

$$V \sim 2z^2 - x^2 - y^2$$





$$\nu_a' = \nu_s - \nu_c' = \frac{g}{2}\nu_c - \nu_c' = \frac{g-2}{2}\nu_c + \nu_m$$

Experimenter's *g*

A real Penning trap has

- distortions in the electrostatic quadrupole
- misalignment of the quadrupole axis and **B**

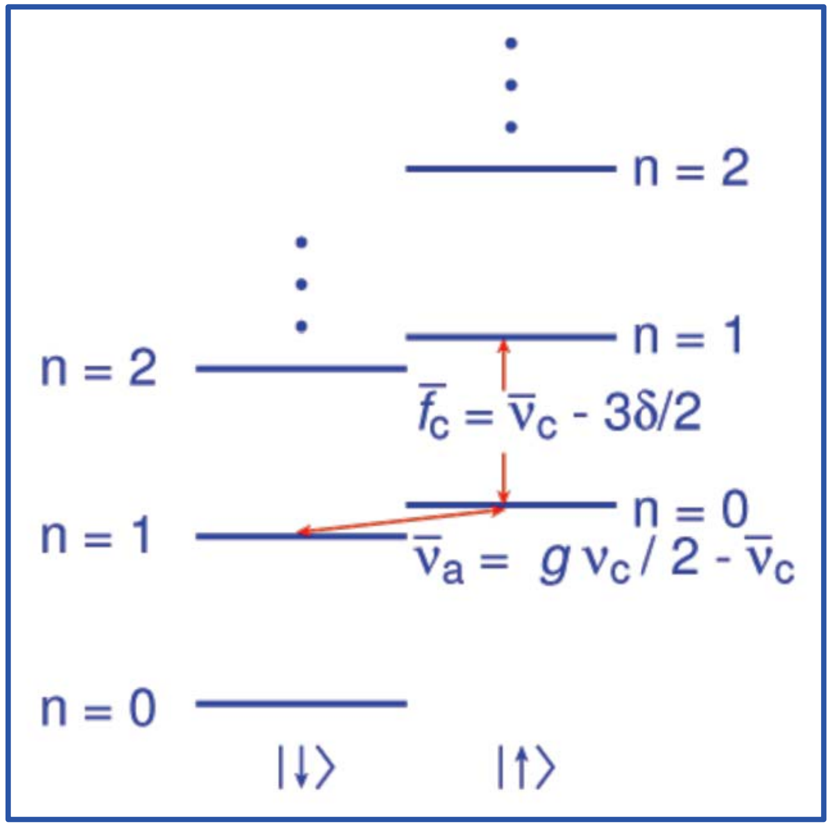
Brown-Gabrielse Invariance Theorem

$$\nu_c = \sqrt{(\bar{\nu}_c)^2 + (\bar{\nu}_z)^2 + (\bar{\nu}_m)^2}$$

L.S. Brown and G. Gabrielse, *Rev. Mod. Phys.* **58**, 233 (1986)

$$\frac{g}{2} \simeq 1 + \frac{\bar{\nu}_a - \frac{\bar{\nu}_z^2}{2\bar{f}_c}}{\bar{f}_c + 3\delta/2 + \frac{\bar{\nu}_z^2}{2\bar{f}_c}} + \frac{\Delta\omega}{\omega}$$

cavity shift



required hierarchy

$$\bar{\nu}_c \gg \bar{\nu}_z \gg \bar{\nu}_m \gg \delta$$

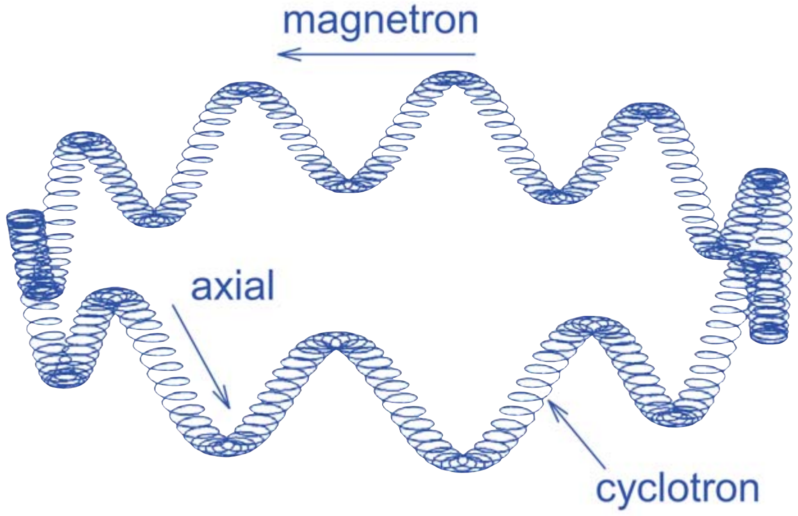
Our Trap Frequencies

$B \sim 5.36$ T

$V_0 \sim 101.4$ V

$d \sim 3.5$ mm (0.14 in)

motion	frequency	damping time
axial	200 MHz	0.2 s
cyclotron	150.0 GHz	5 s
spin	150.2 GHz	2 yr
magnetron	133 kHz	4 Gyr

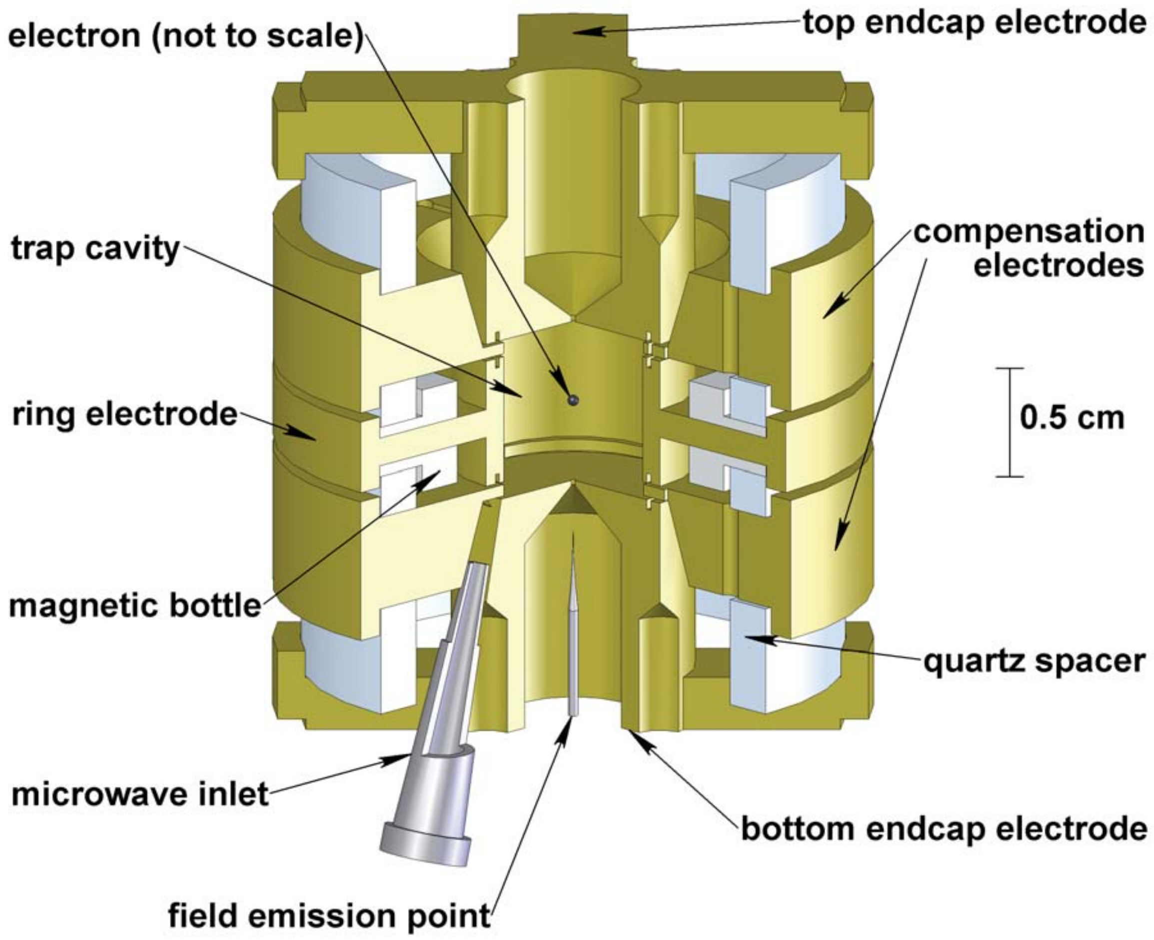


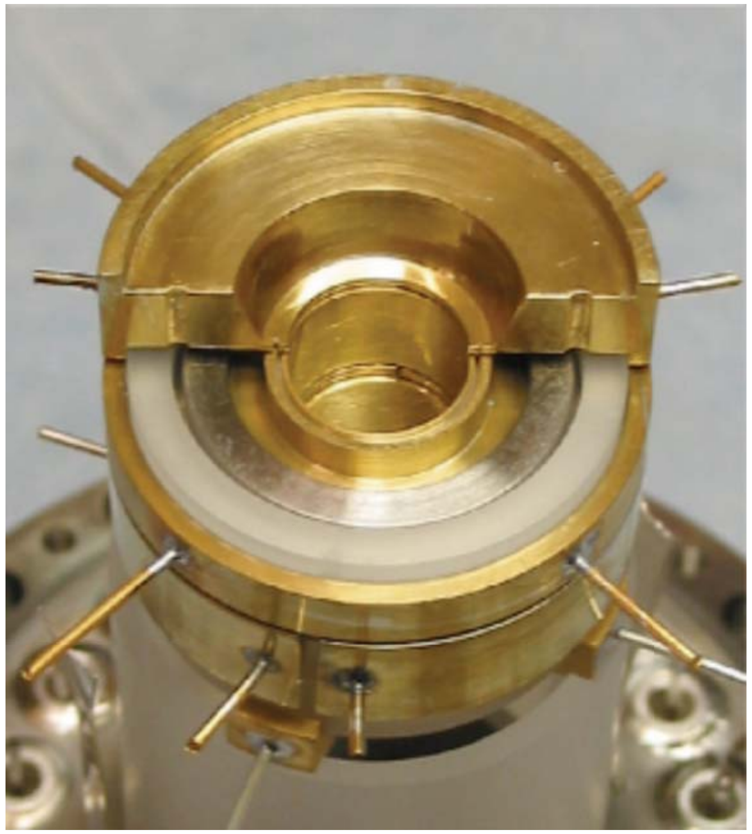
hierarchy satisfied

$$\bar{\nu}_c \gg \bar{\nu}_z \gg \bar{\nu}_m \gg \delta$$

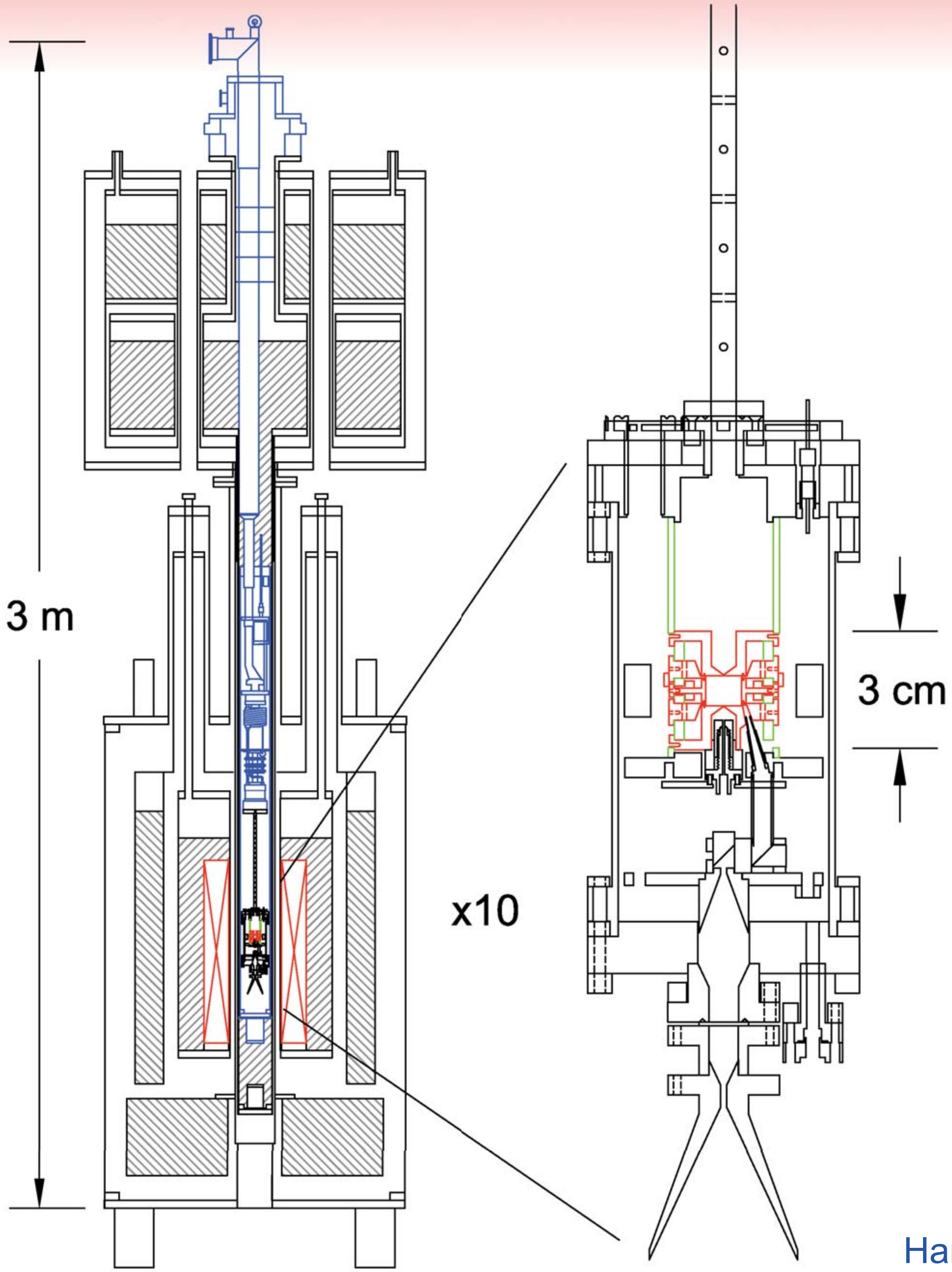
$\delta \sim 180$ Hz

Our Trap

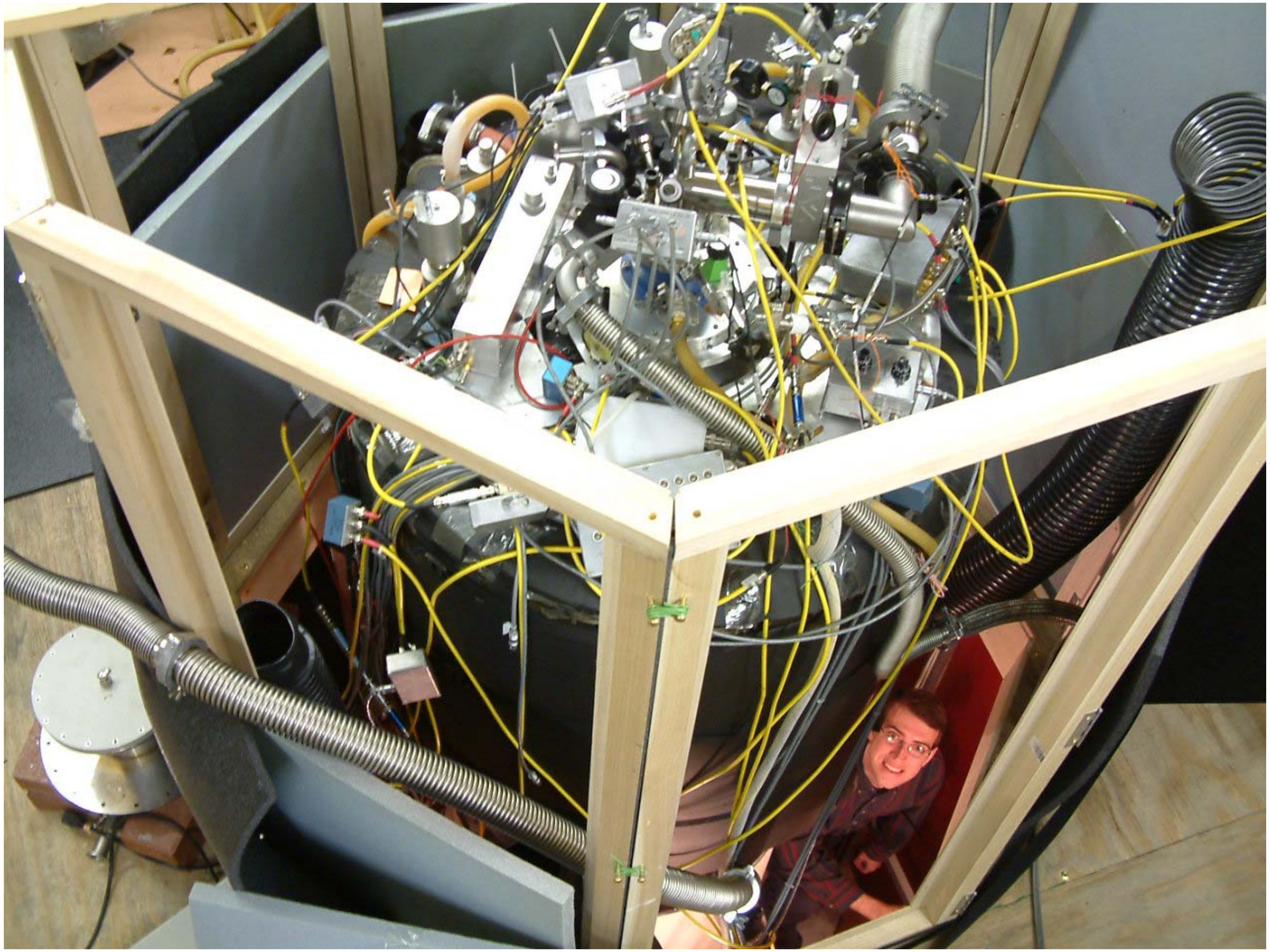

electron (not to scale)
top endcap electrode
trap cavity
compensation electrodes
ring electrode
0.5 cm
magnetic bottle
quartz spacer
microwave inlet
bottom endcap electrode
field emission point



The Whole Apparatus



A Tabletop Experiment



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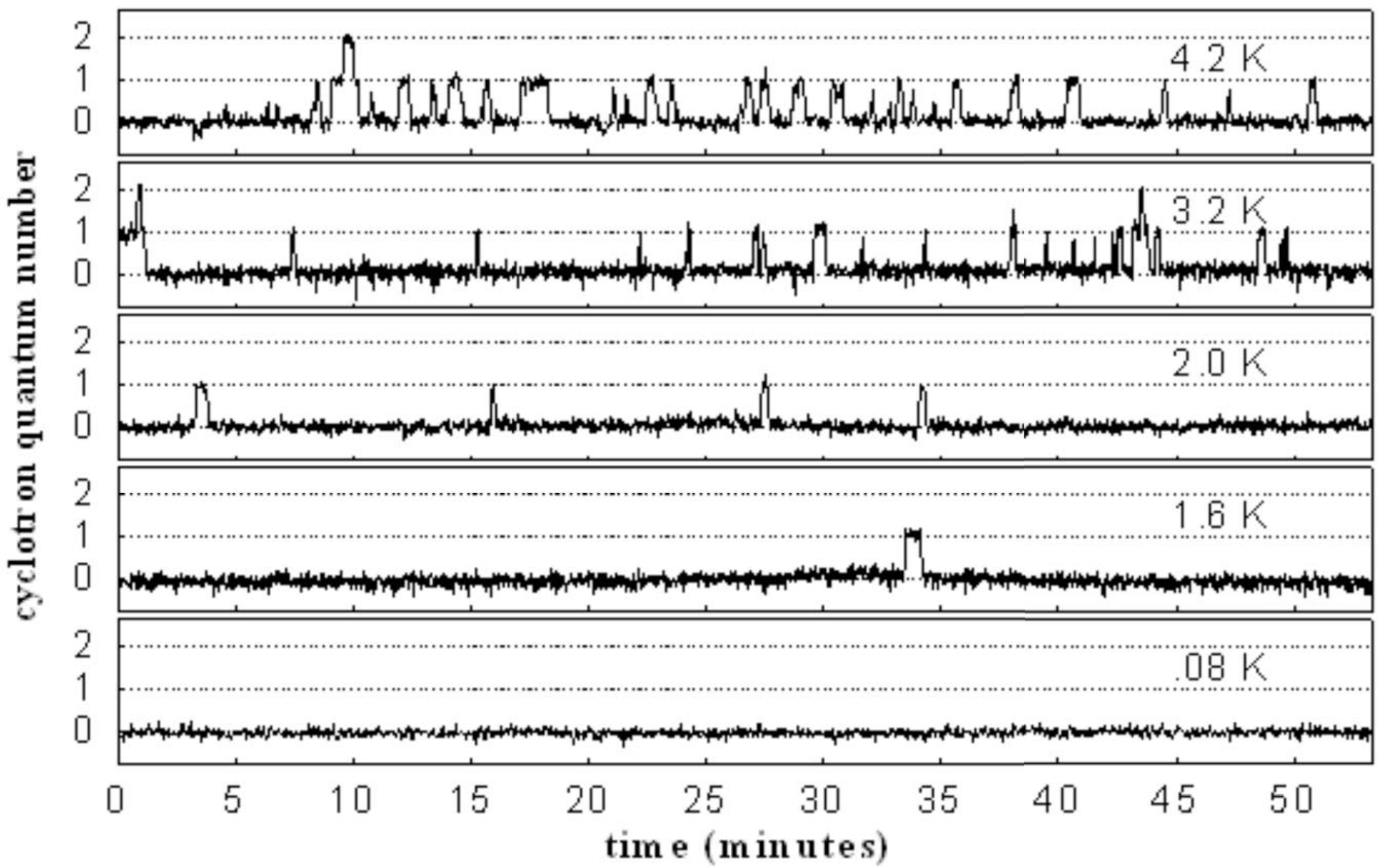
Low Temperatures

Cool the blackbody photons (⟨n⟩ << 1)

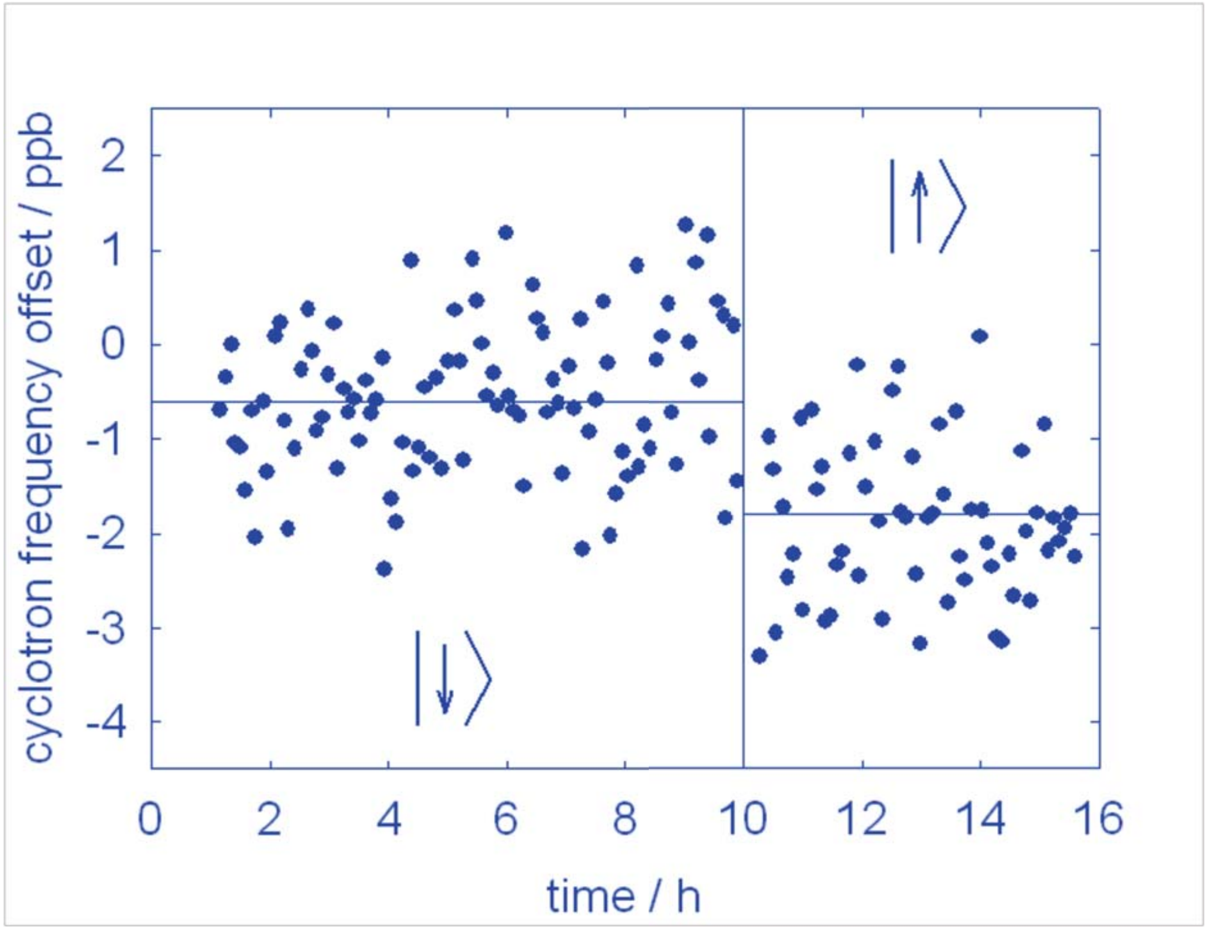
Allows quantum jump spectroscopy

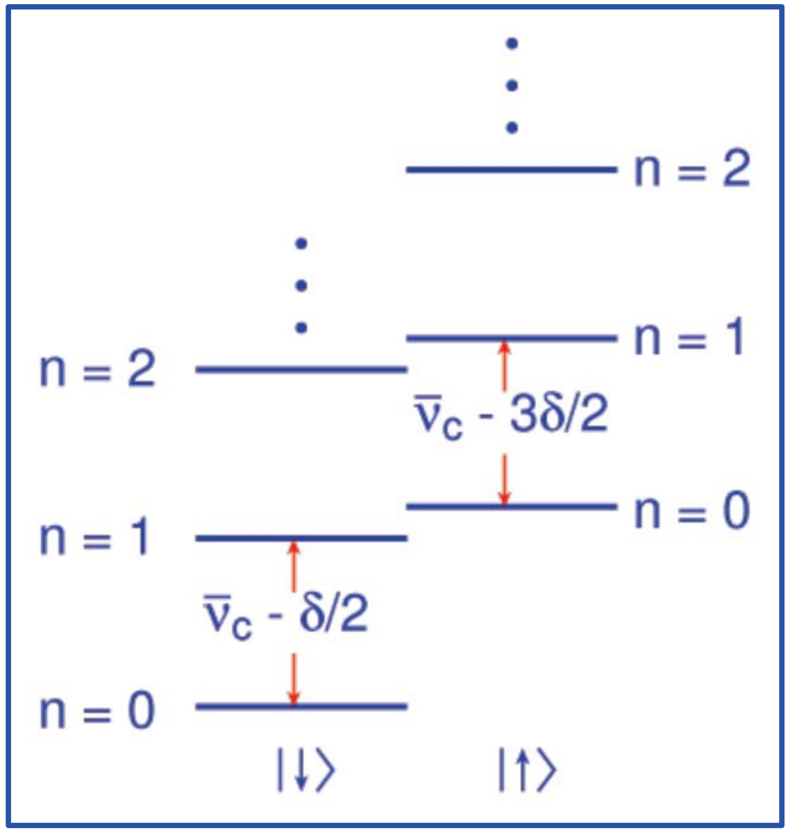


S.Peil and G.Gabrielse, *Phys. Rev. Lett.* **83**, 1287 (1999)

Low Temperatures

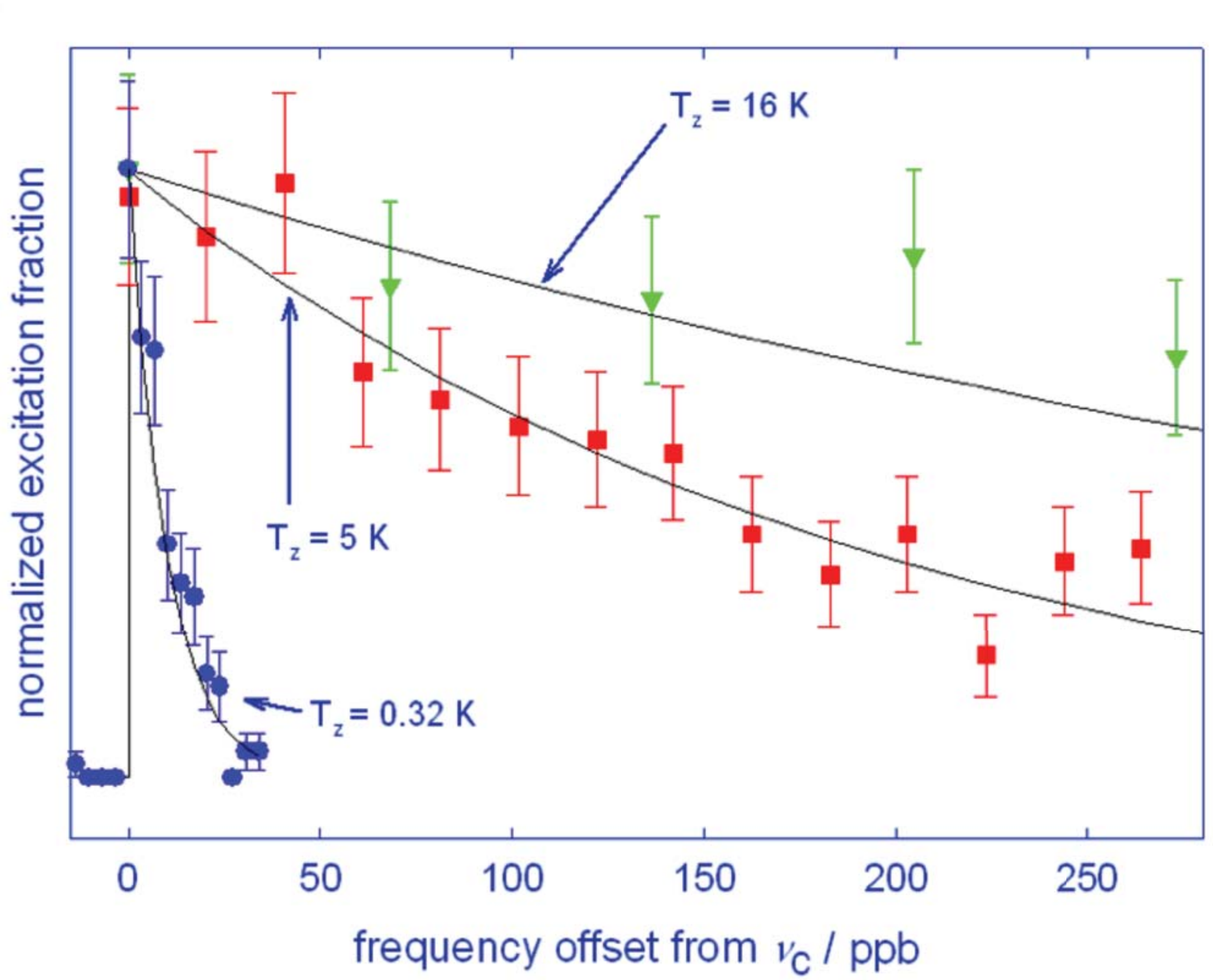
Exact relativistic shifts





Low Temperatures

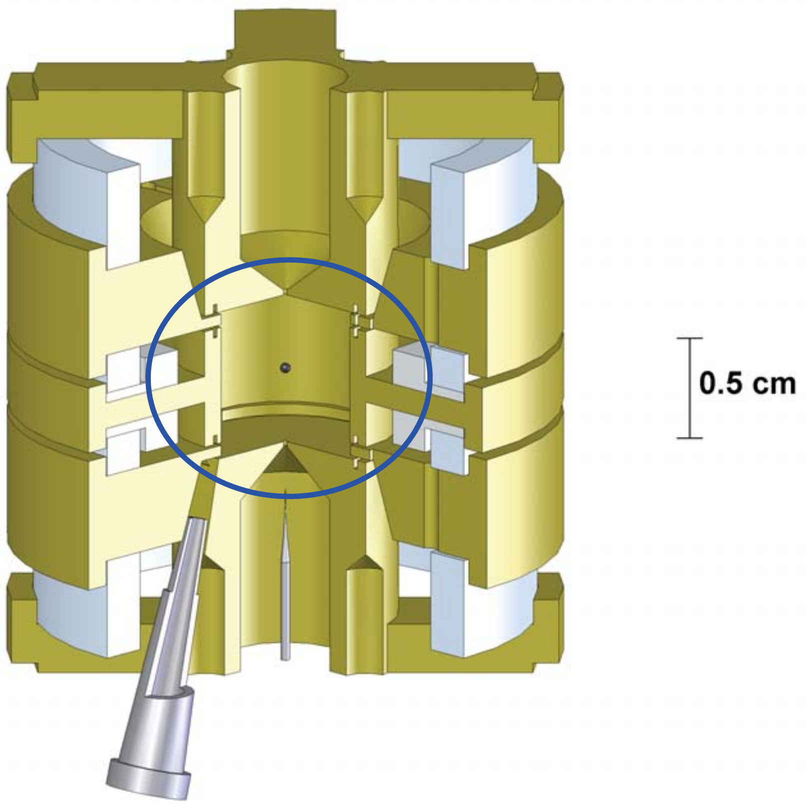
Narrow line widths

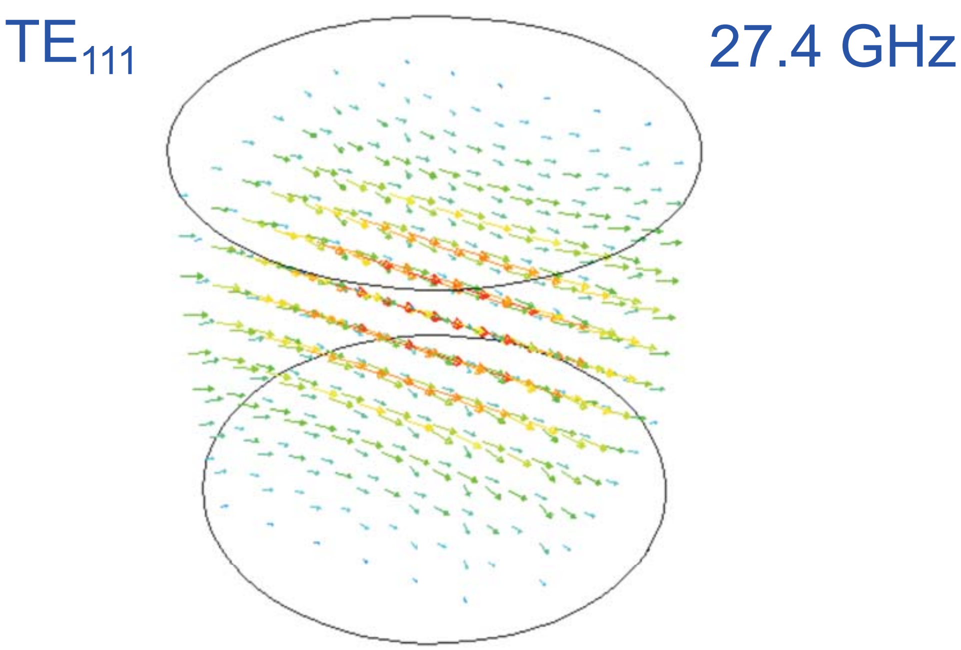


Cavity Control

The radiation modes of a cylindrical cavity are

- Well understood
- Resonant near ν_c = 150 GHz (λ ~ 2 mm)
- Coupled to the cyclotron motion if the geometry is right



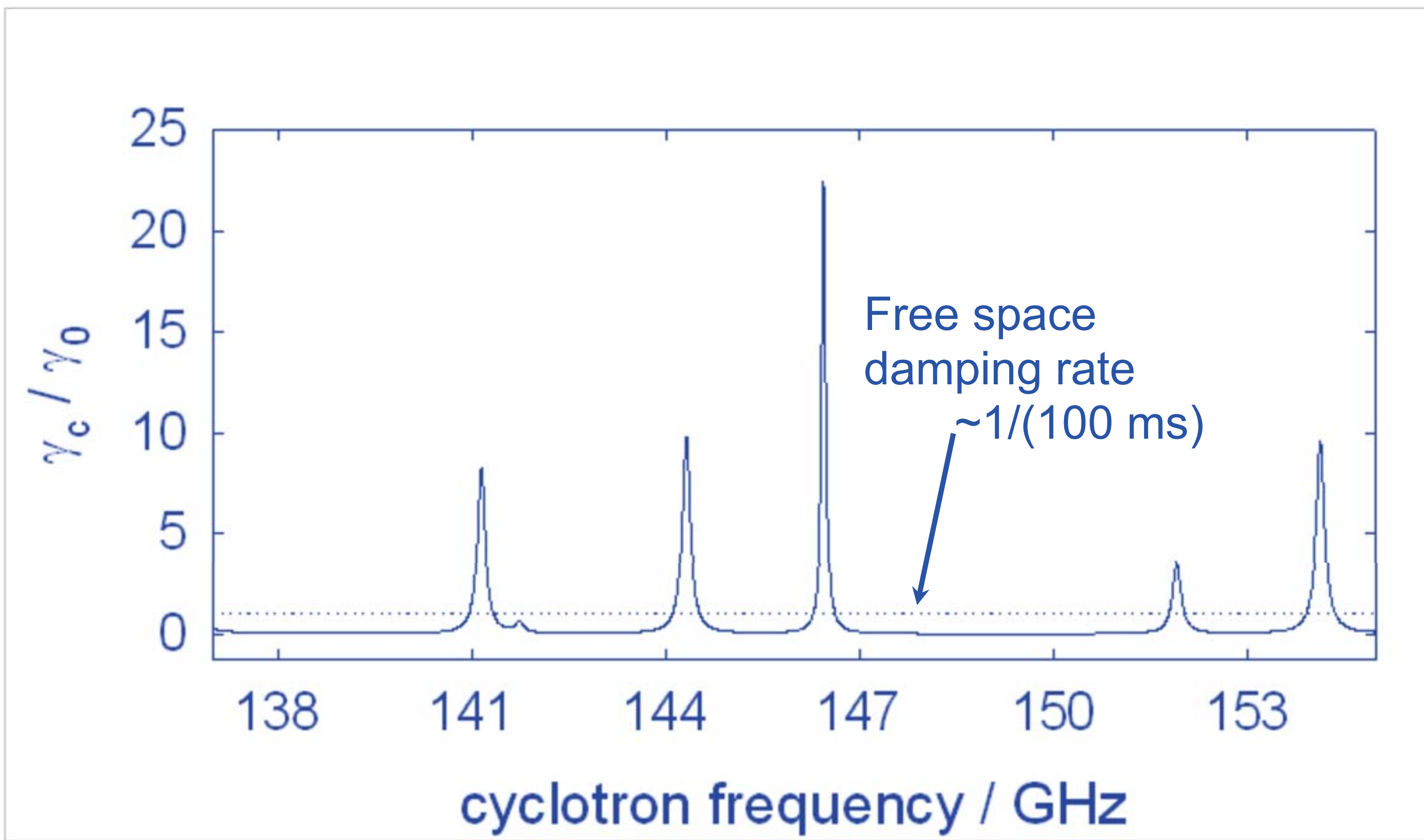


J.D. Jackson, *Classical Electrodynamics*, 3rd Ed., Sect. 8.7

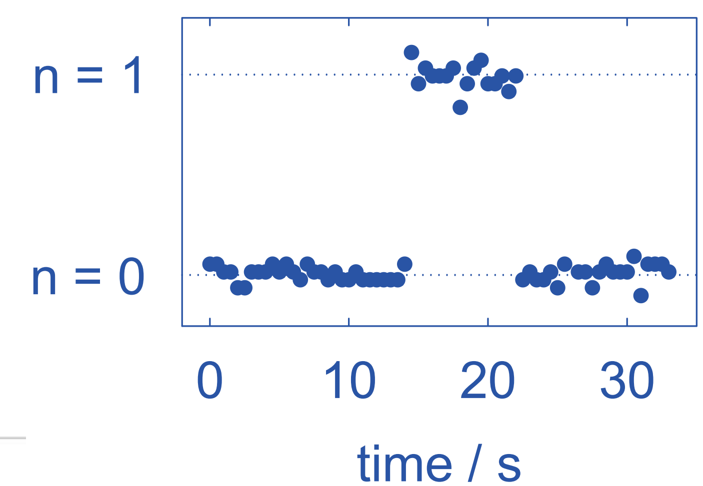
L.S. Brown *et al.*, *Phys. Rev. A* **32**, 3204 (1985)

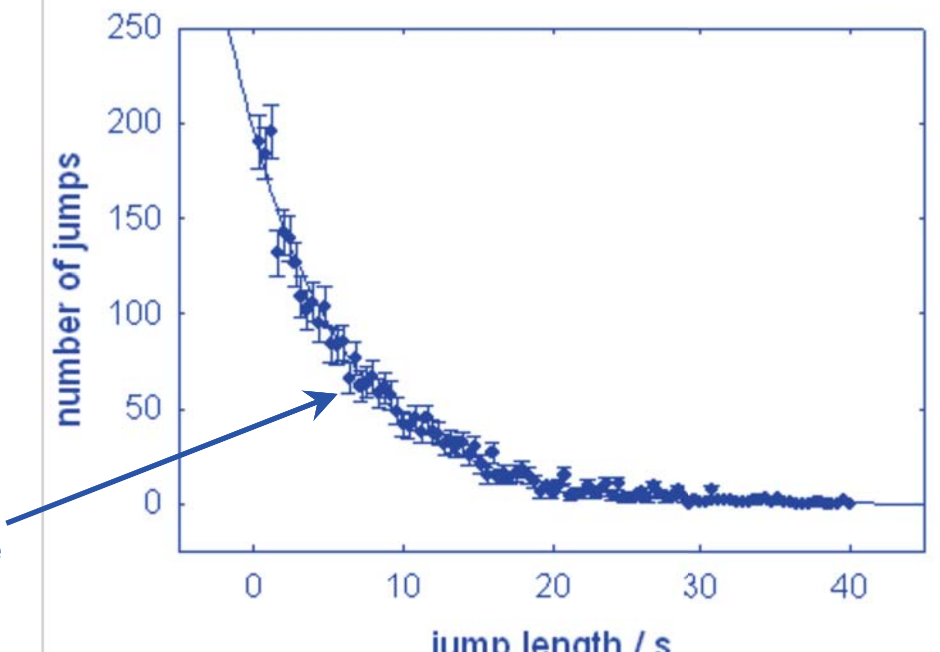
Cavity Control

Control the cyclotron damping rate, γ_c



Inhibited spontaneous emission

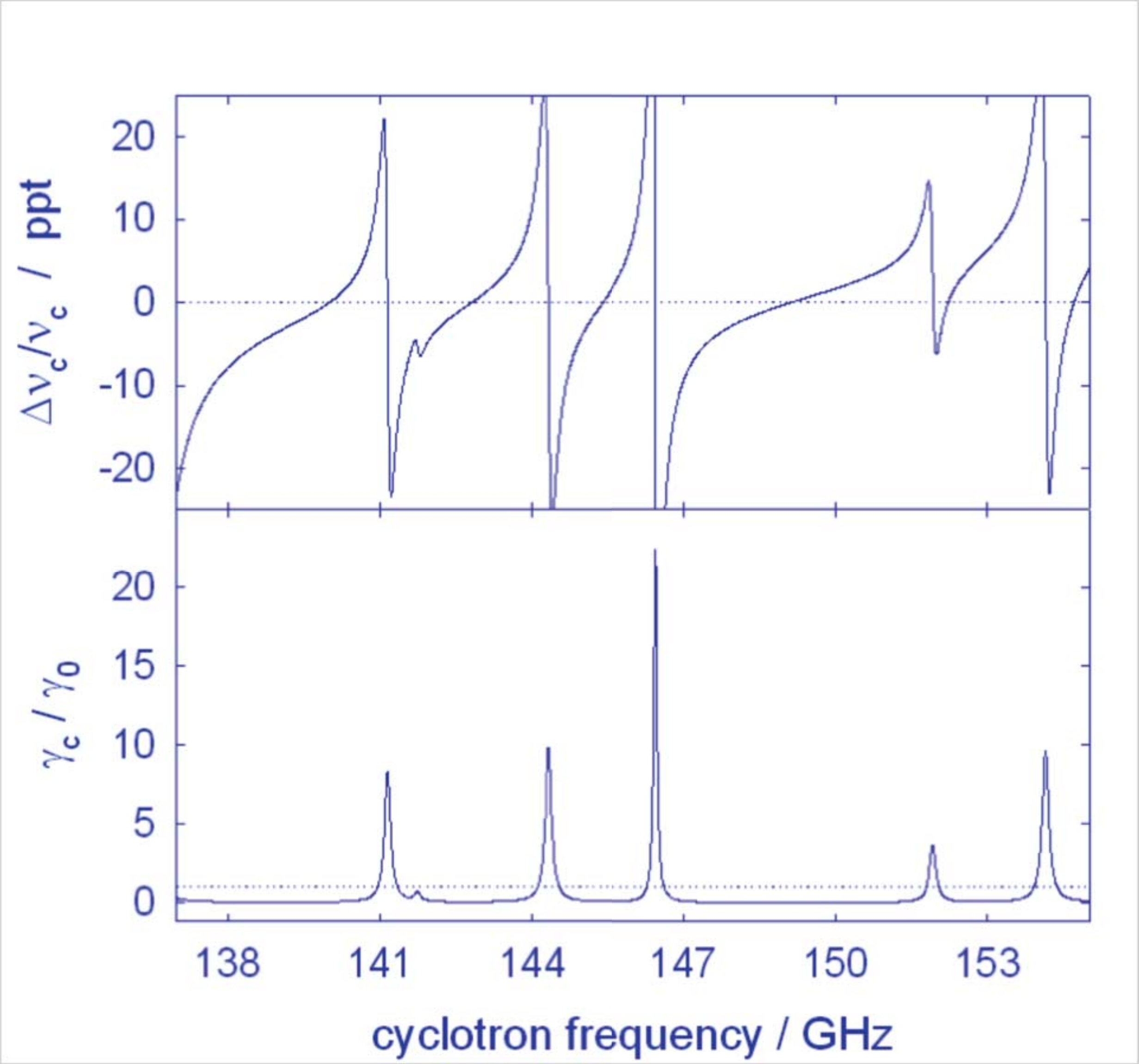




$\tau = (6.70 \pm 0.18)\ \mathrm{s} = 73.6 \times \tau_{\text{free space}}$

Cavity Control

Shift the cyclotron frequency



$$\frac{\Delta g}{g} = -\frac{\Delta \nu_c}{\nu_c}$$

Details later…

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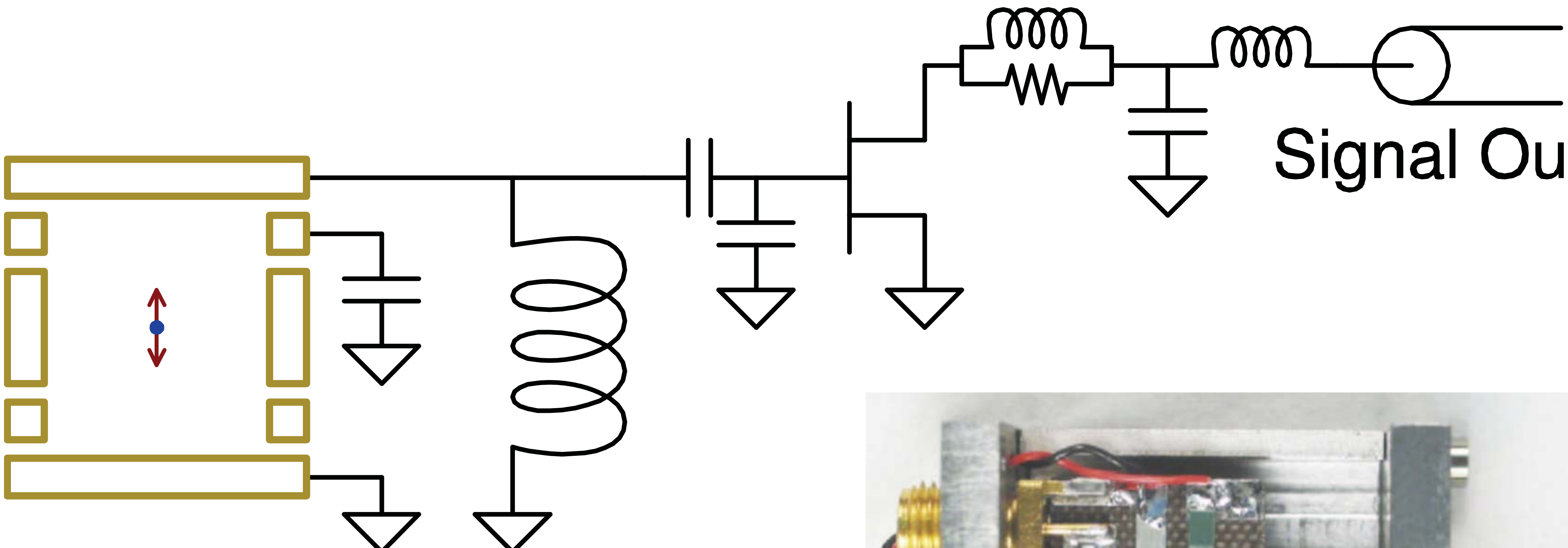
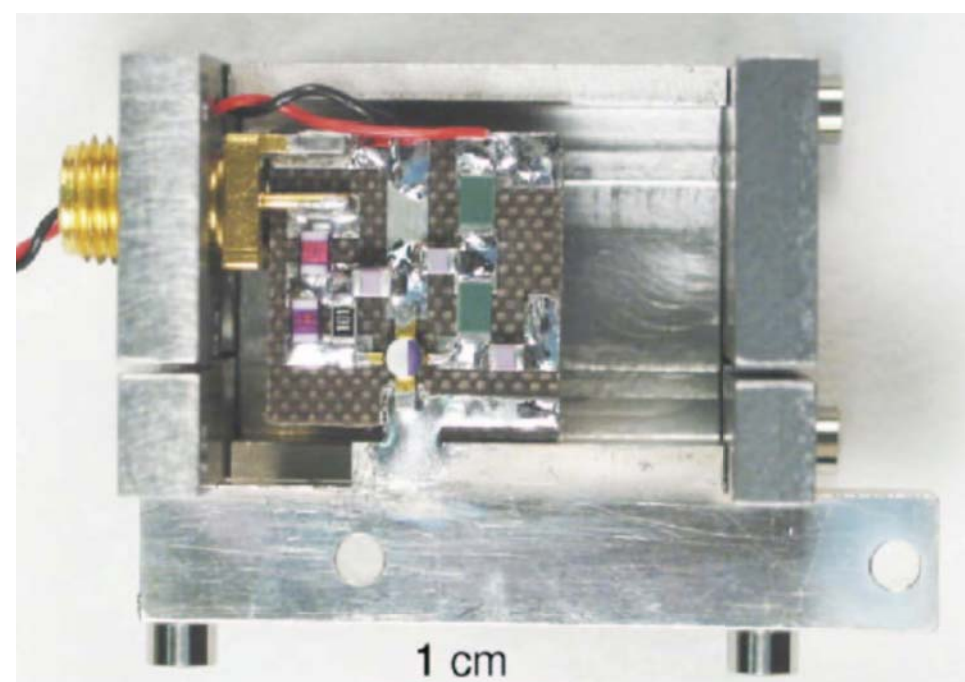
Determining *g*

$$\frac{g}{2} \simeq 1 + \frac{\bar{\nu}_a - \frac{\bar{\nu}_z^2}{2\bar{f}_c}}{\bar{f}_c + 3\delta/2 + \frac{\bar{\nu}_z^2}{2\bar{f}_c}} + \frac{\Delta\omega}{\omega}$$

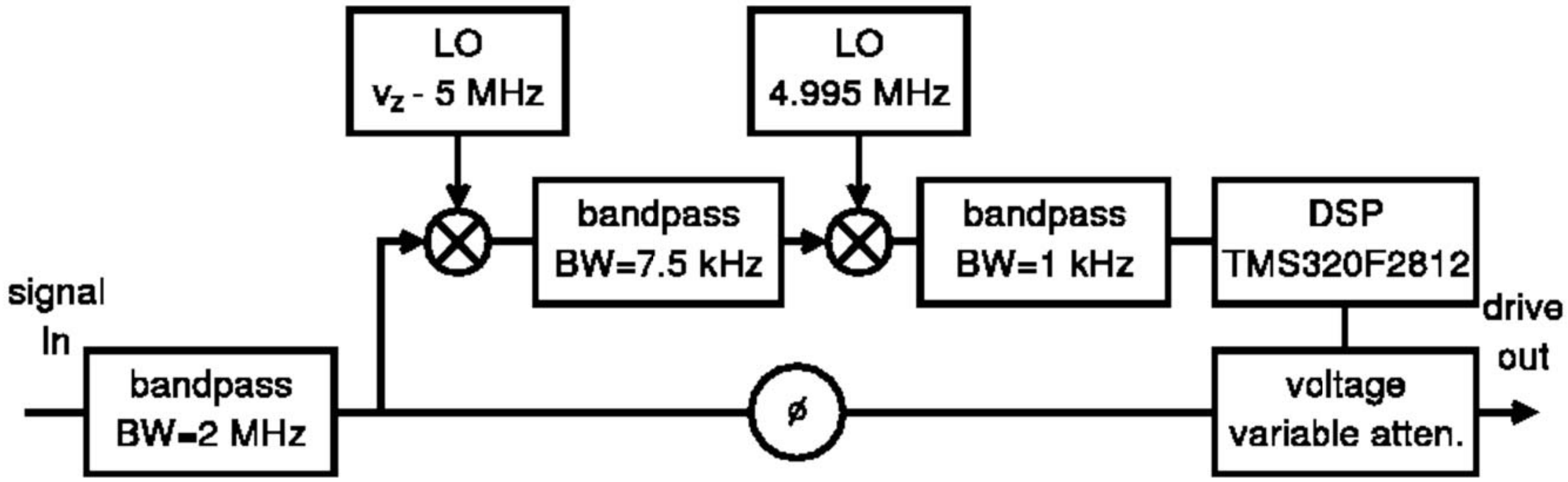
- Measure $\bar{f}_c$, $\bar{\nu}_a$, $\bar{\nu}_z$
- Calculate δ
- Calculate $\Delta\omega/\omega$

Axial Detection

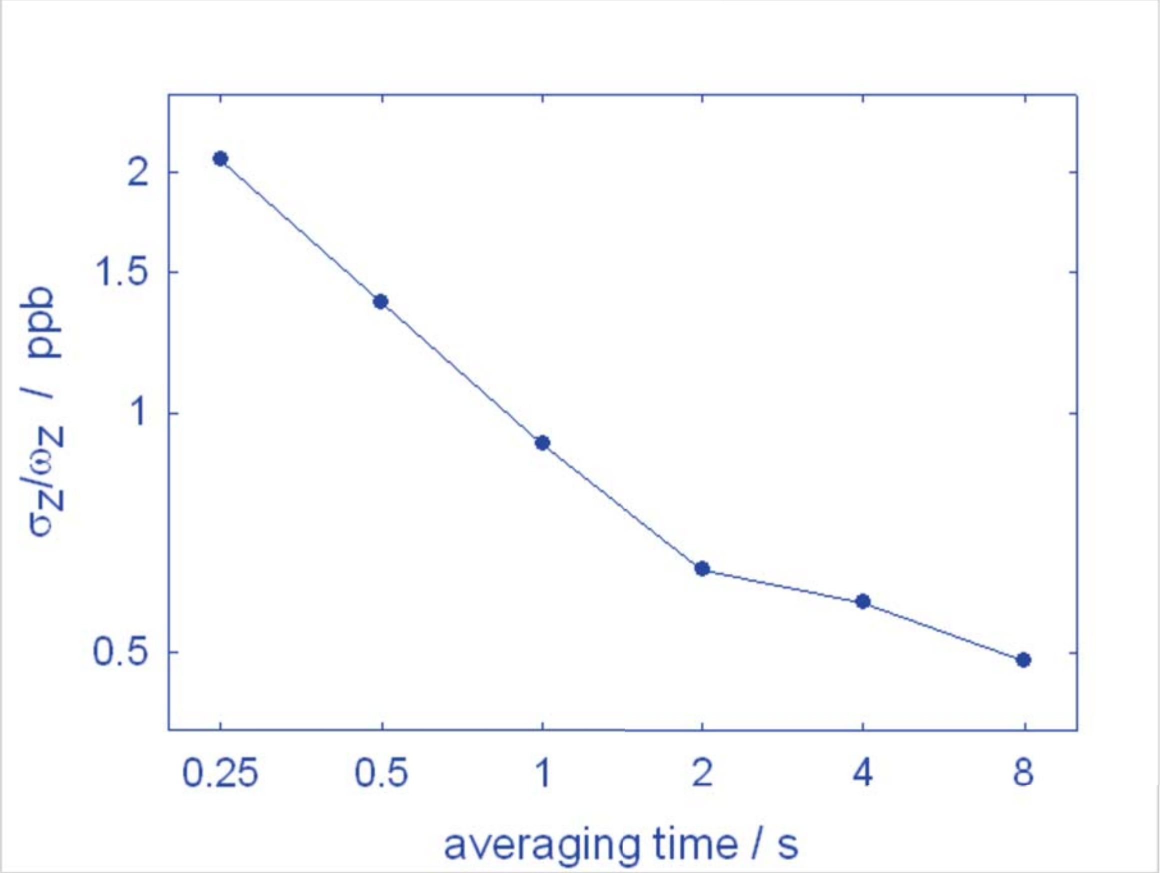




Self Excited Oscillator

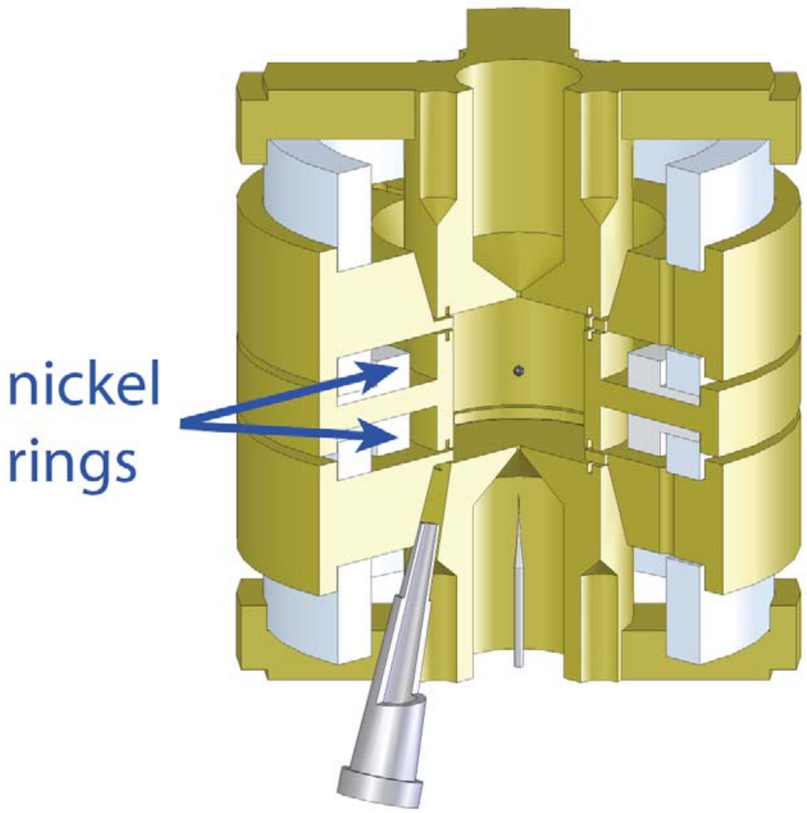


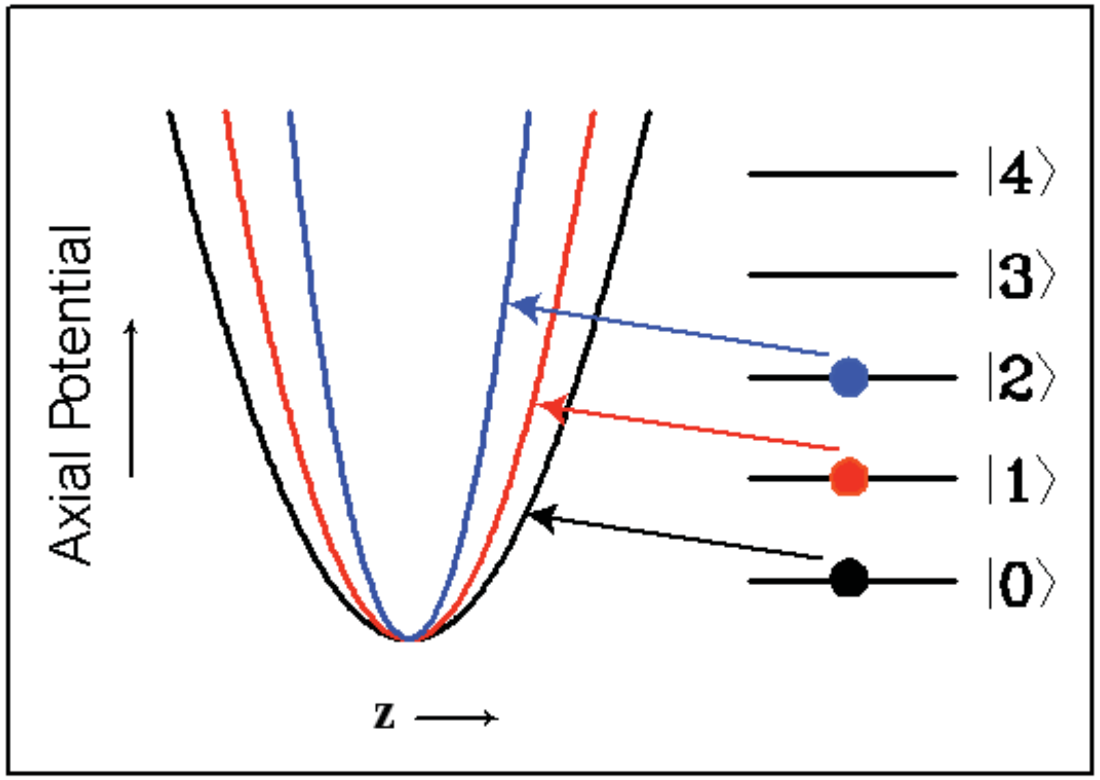
- First single-particle SEO
- Enhanced S/N



B. D'Urso, R. Van Handel, B. Odom, D. Hanneke, and G. Gabrielse,
Phys. Rev. Lett. **94**, 113002 (2005)

Cyclotron/Spin Detection





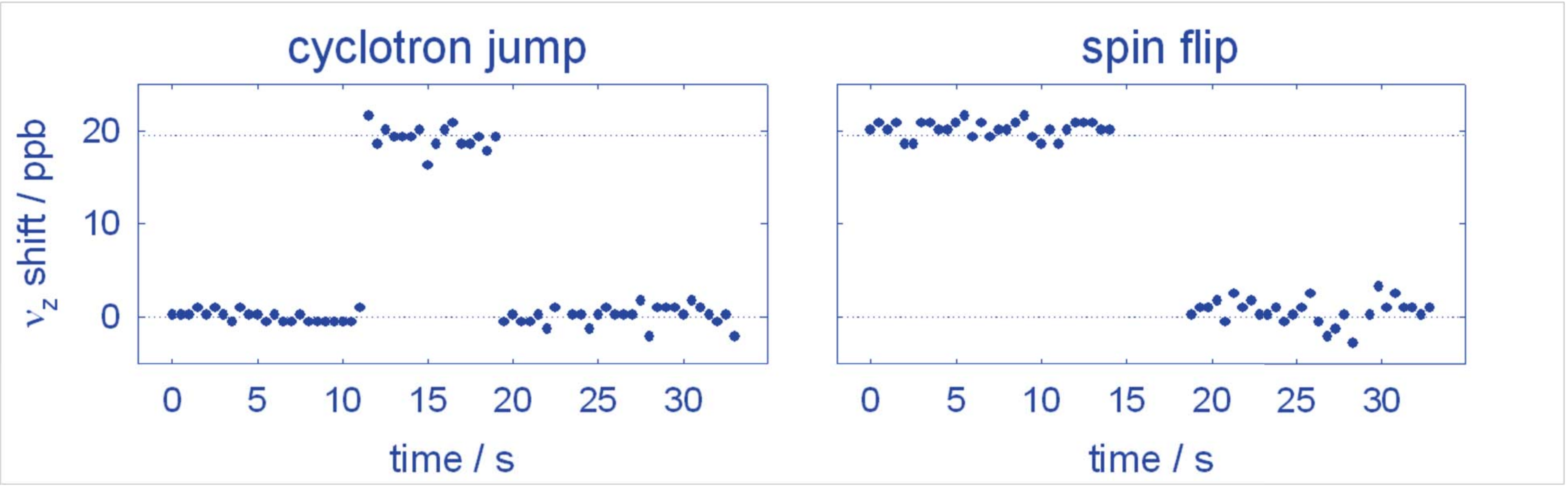
$$\Delta\vec{B} = B_2\left[\left(z^2 - \rho^2/2\right)\hat{z} - z\rho\hat{\rho}\right]$$

$$H_{z0} + H_z' = \frac{1}{2}m\omega_{z0}^2 z^2 - \mu_{s,c}B_2 z^2$$

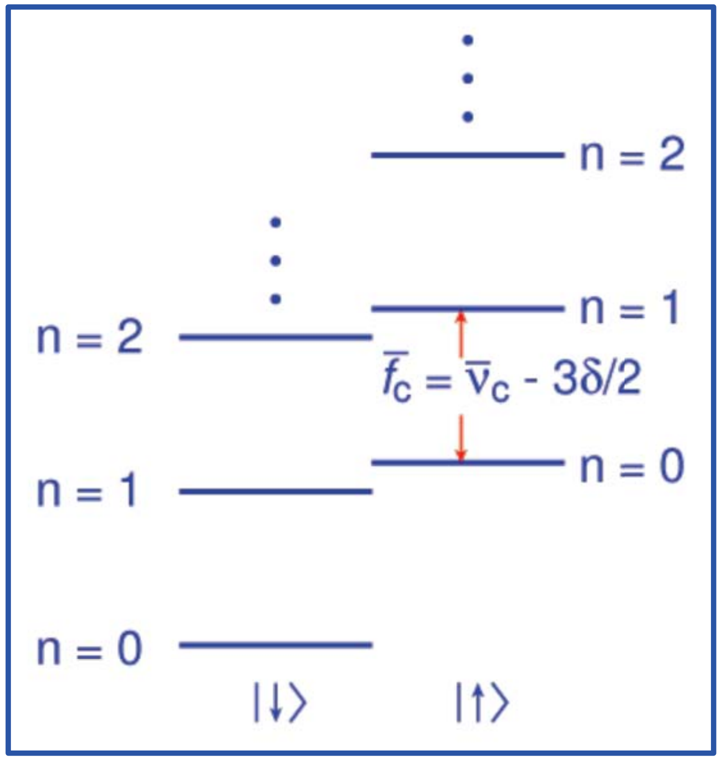
$$\frac{\Delta\nu_z}{\nu_z} \approx 2\times 10^{-8}\left(\frac{g}{2}m_s + n\right)$$

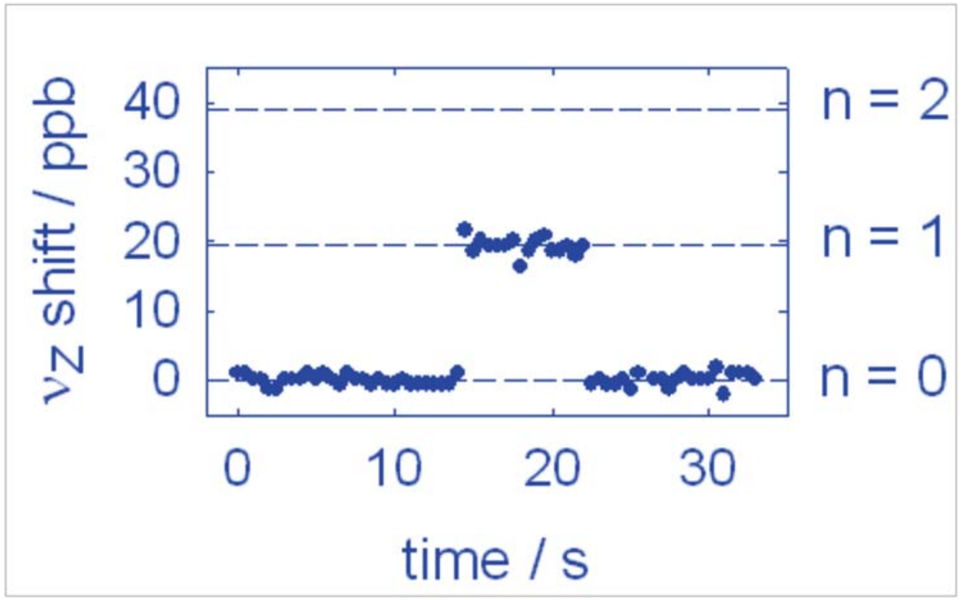
QND measurement: $[H_c, H_z'] = 0$

Quantum Leaps

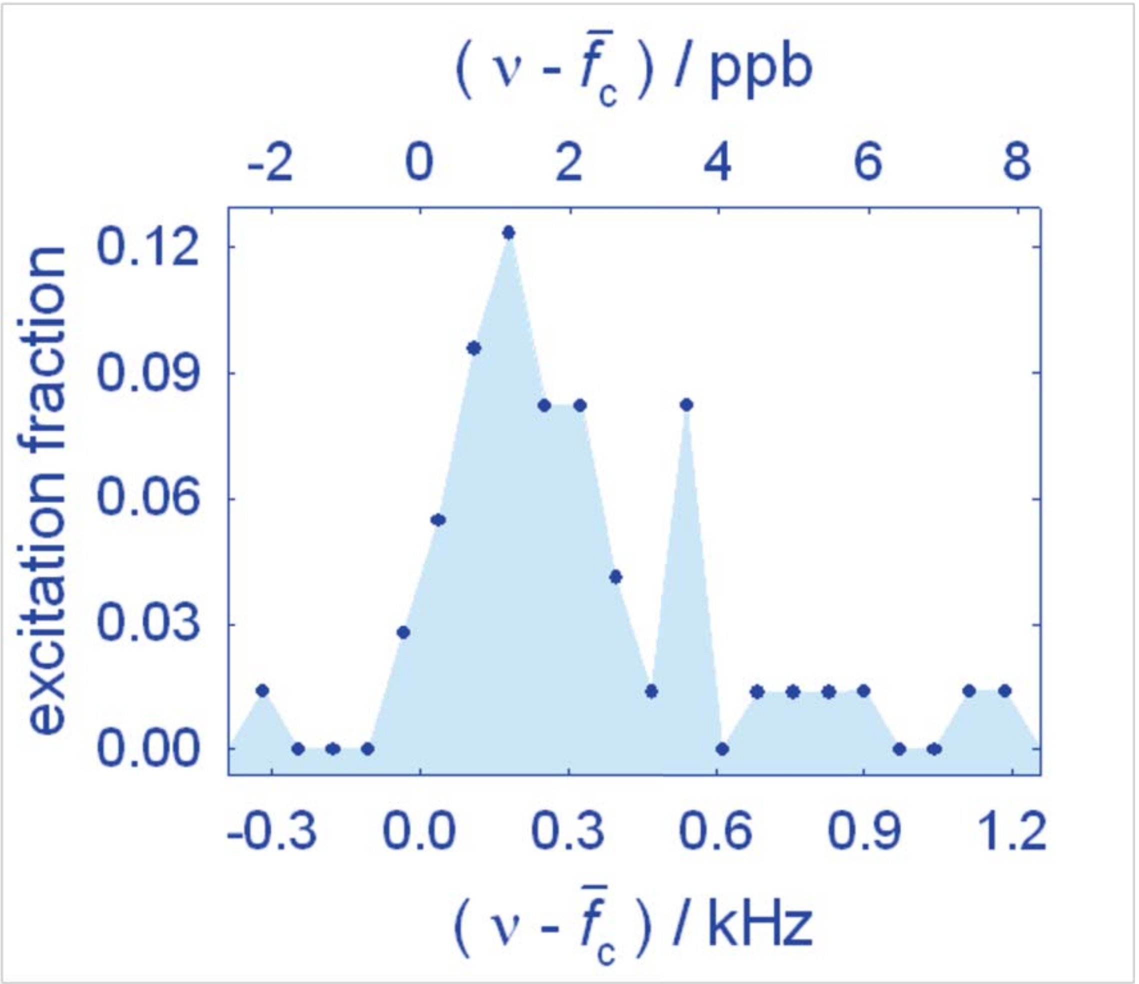


Cyclotron Procedure

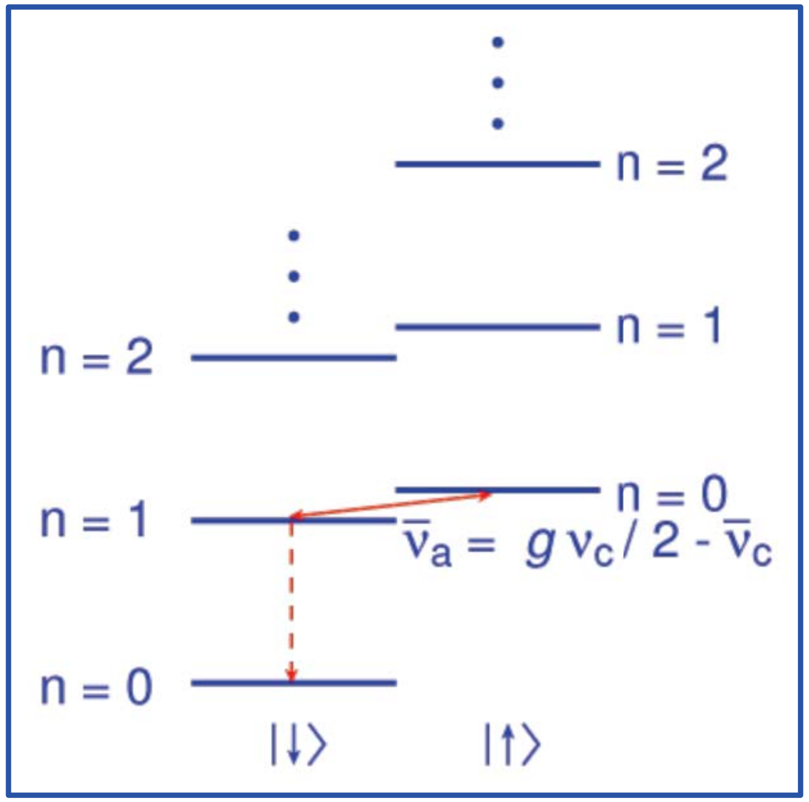




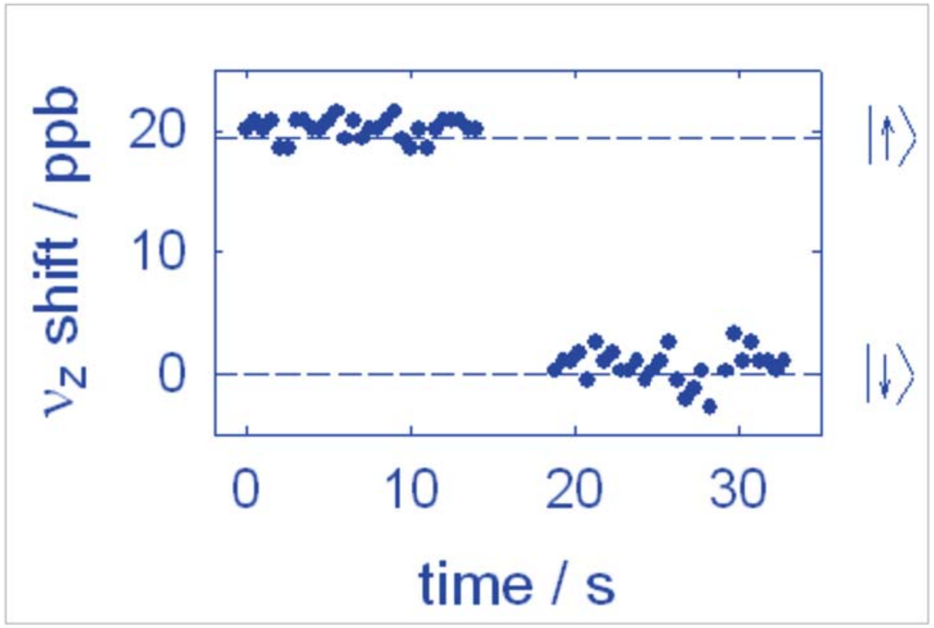
- With the e^- in the $|0,\uparrow\rangle$ state, pulse the cyclotron drive (150 GHz)
- Look for excitations to n = 1
- Make a histogram of excitations versus frequency

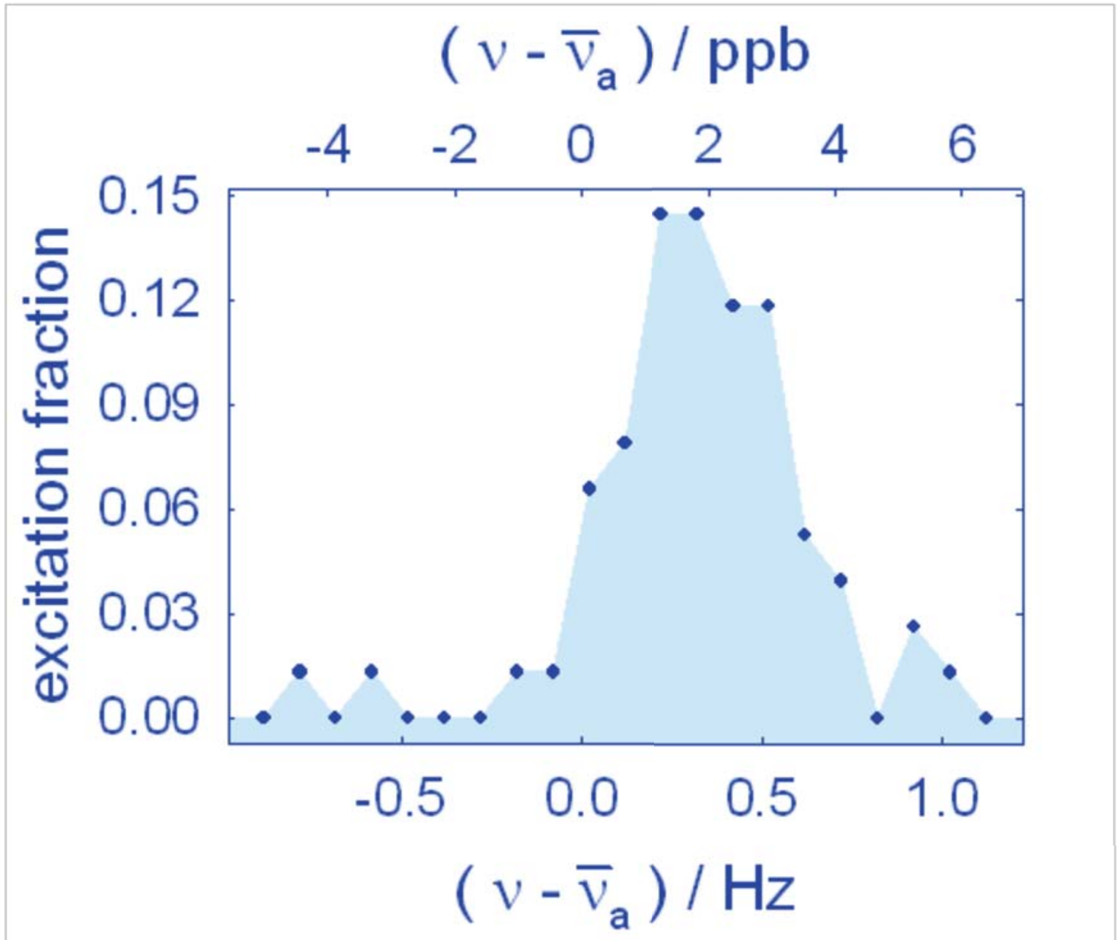


Anomaly Procedure

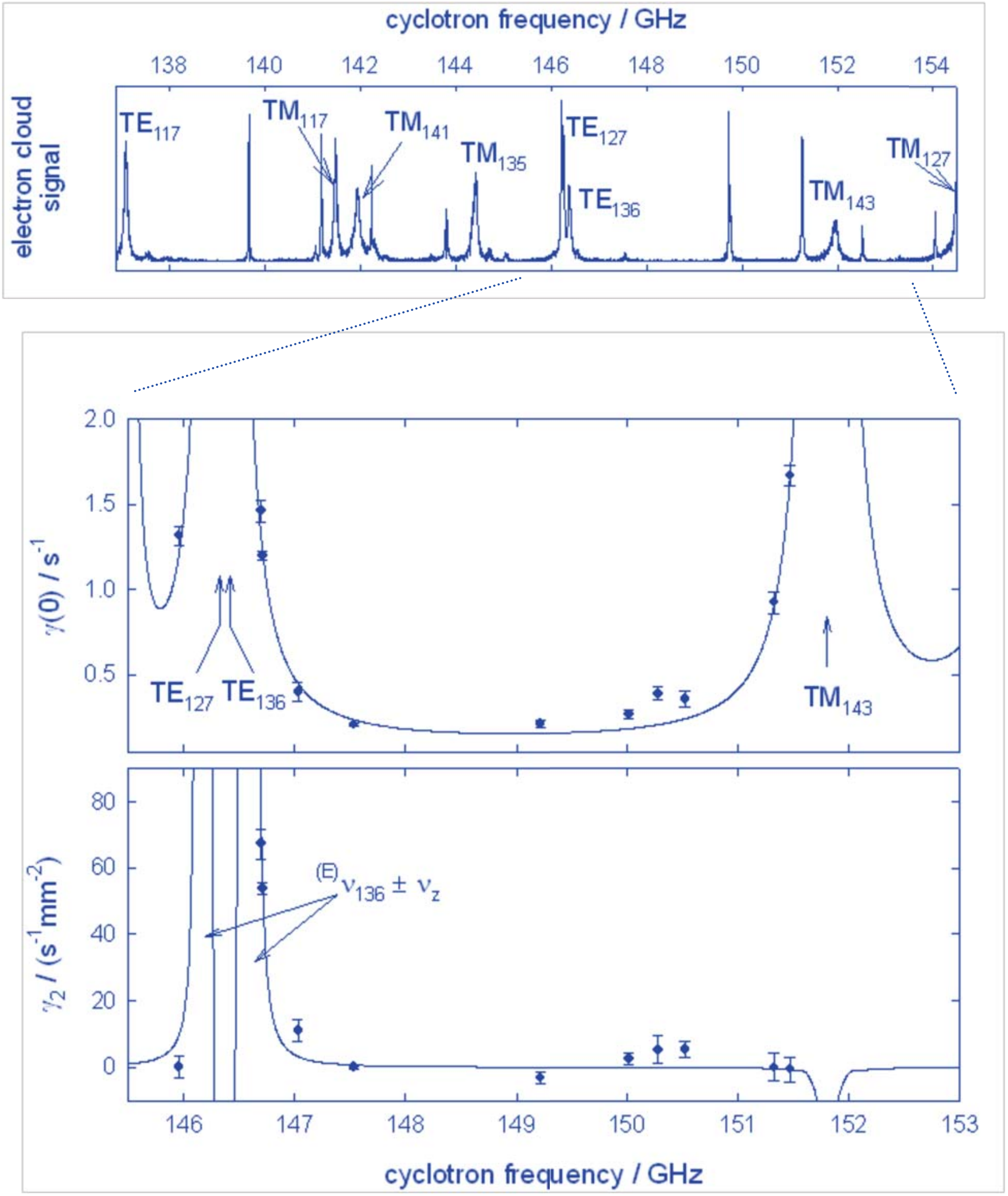


- With the e^- in the $|0,\uparrow\rangle$ state, pulse the anomaly drive (174 MHz) to move the e^- through the gradient $\Delta\vec{B} \sim z\rho\hat{\rho}$
- Look for a decay to $|0,\downarrow\rangle$ (slow!)
- Make a histogram of spin flips versus frequency
- Pump the electron back to $|0,\uparrow\rangle$ state before each measurement.



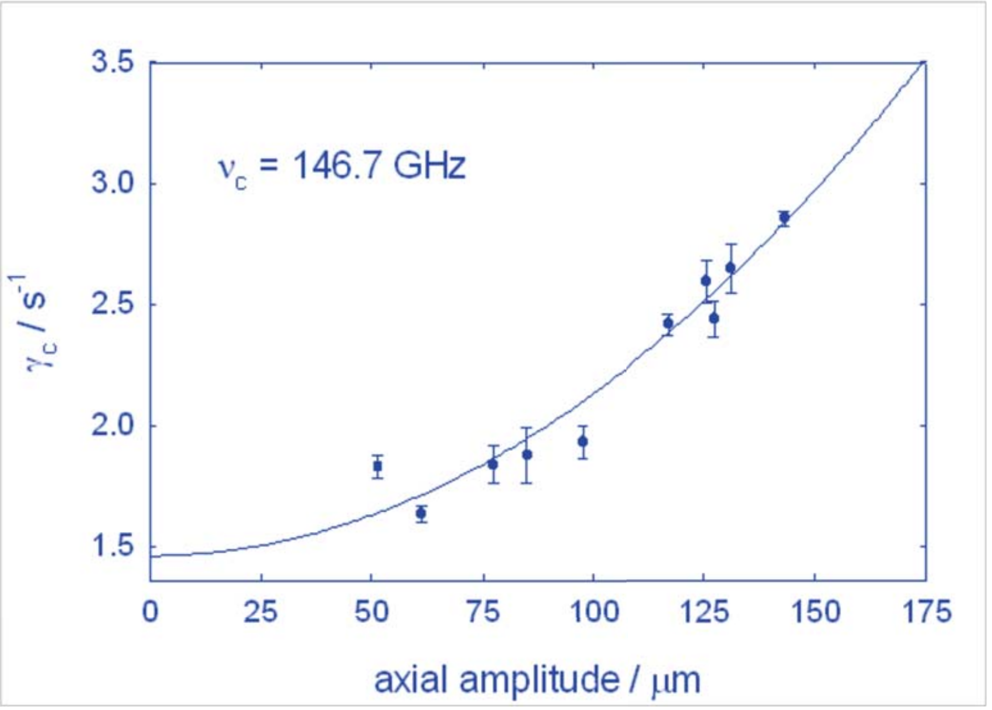


Determining the Cavity Shifts



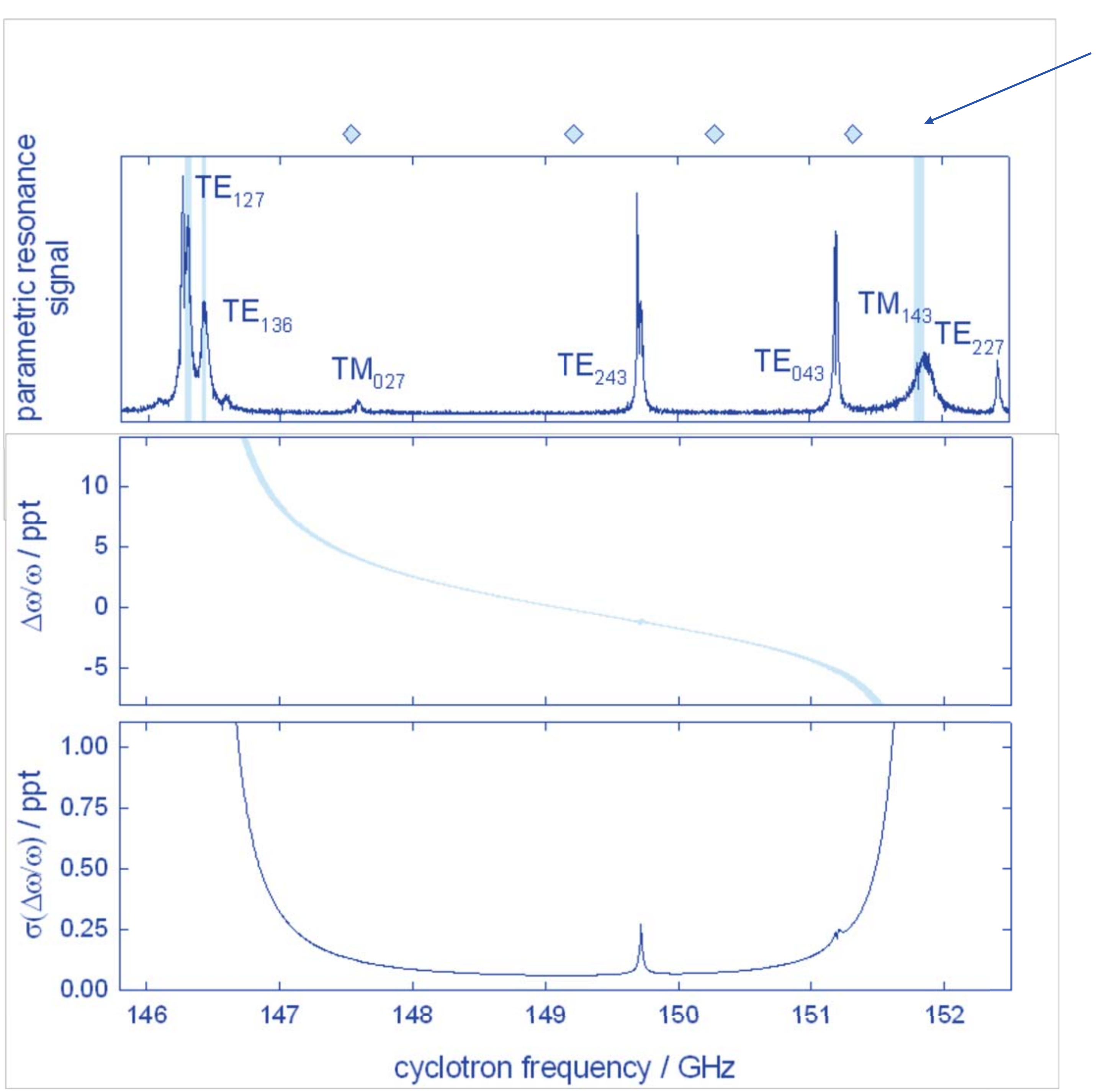
Two mode-detection techniques

- A synchronized cloud of electrons
- The single electron itself



Synchronized electron technique:
J. Tan and G Gabrielse, *Phys. Rev. Lett.* **67**, 3090 (1991)

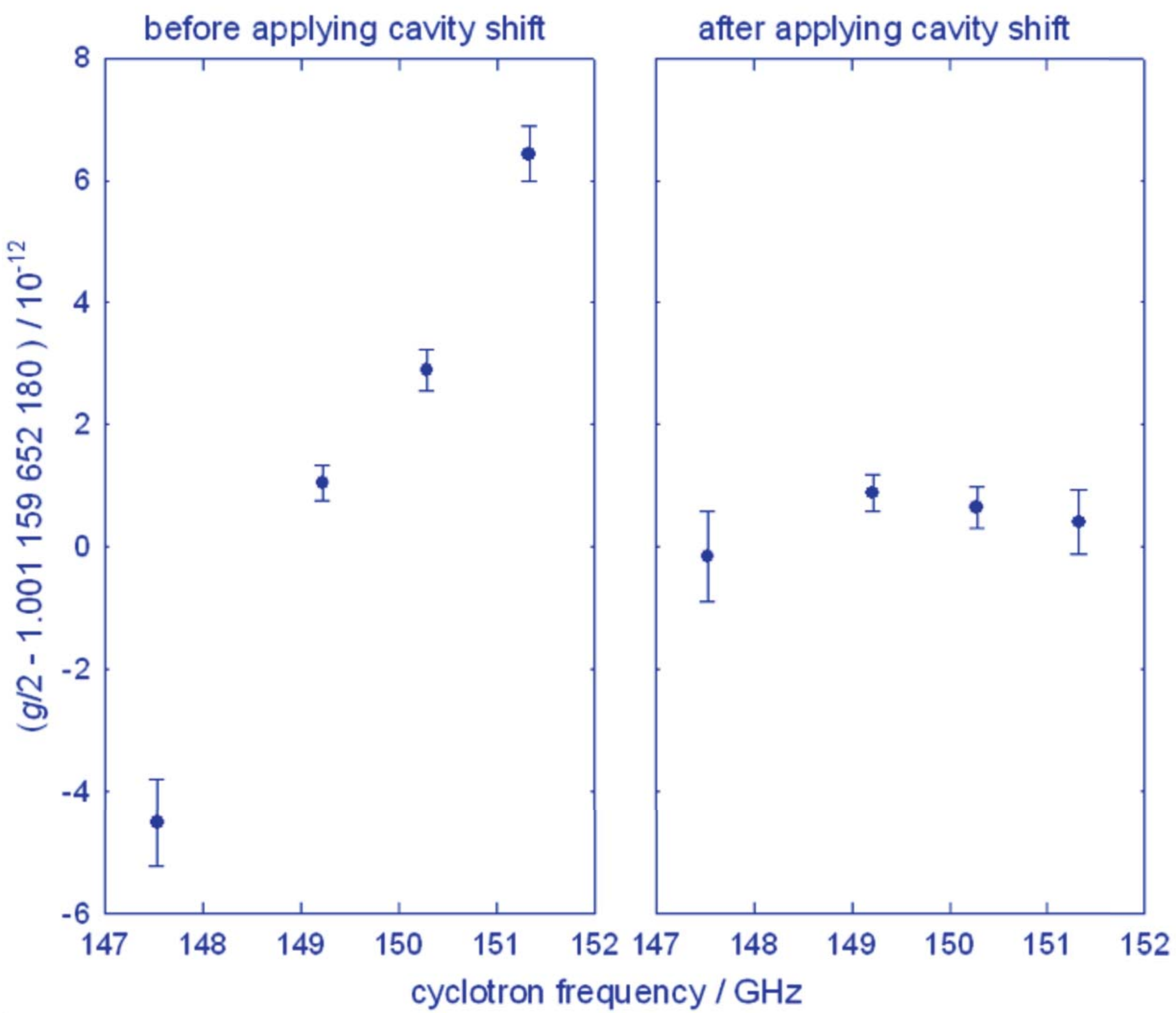
Determining the Cavity Shifts



- Determine the locations of the coupled modes
- Calculate the *g* value shift
- Uncertainty in mode location → uncertainty in *g*

L.S. Brown *et al.*, *Phys. Rev. A* **32**, 3204 (1985)

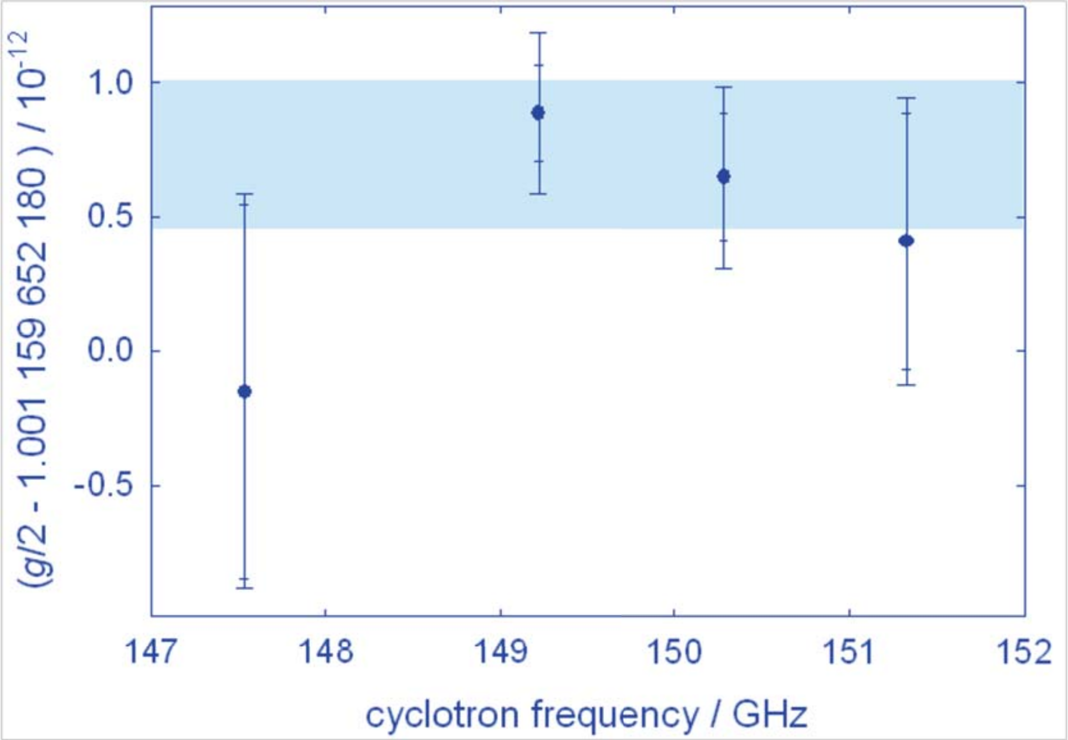
Applying the Cavity Shifts



Outline

Uncertainties

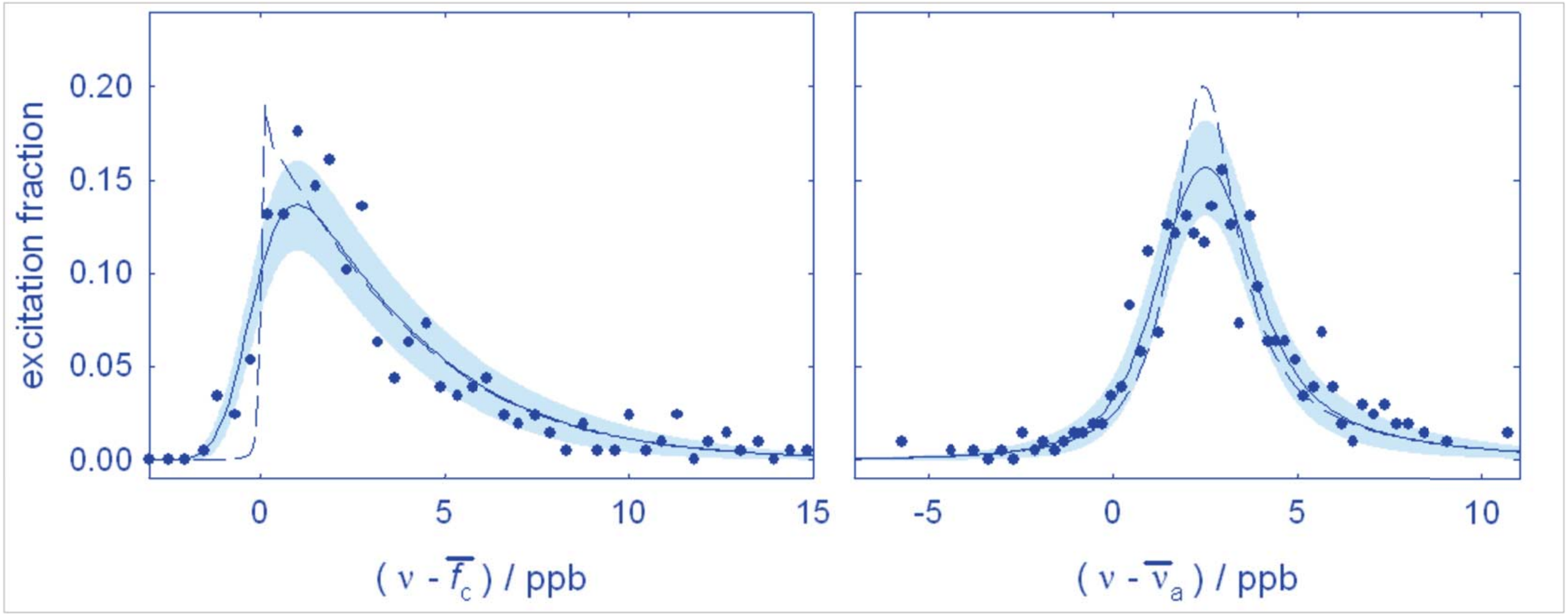
g/2 = 1.001 159 652 180 73 (28) [0.28 ppt]



Uncertainties for *g* in parts-per-trillion.

ν_c / GHz =	147.5	149.2	150.3	151.3
Statistics	0.39	0.17	0.17	0.24
Cavity shift	0.13	0.06	0.07	0.28
Uncorrelated lineshape model	0.56	0.00	0.15	0.30
Correlated lineshape model	0.24	0.24	0.24	0.24
Total	**0.73**	**0.30**	**0.34**	**0.53**

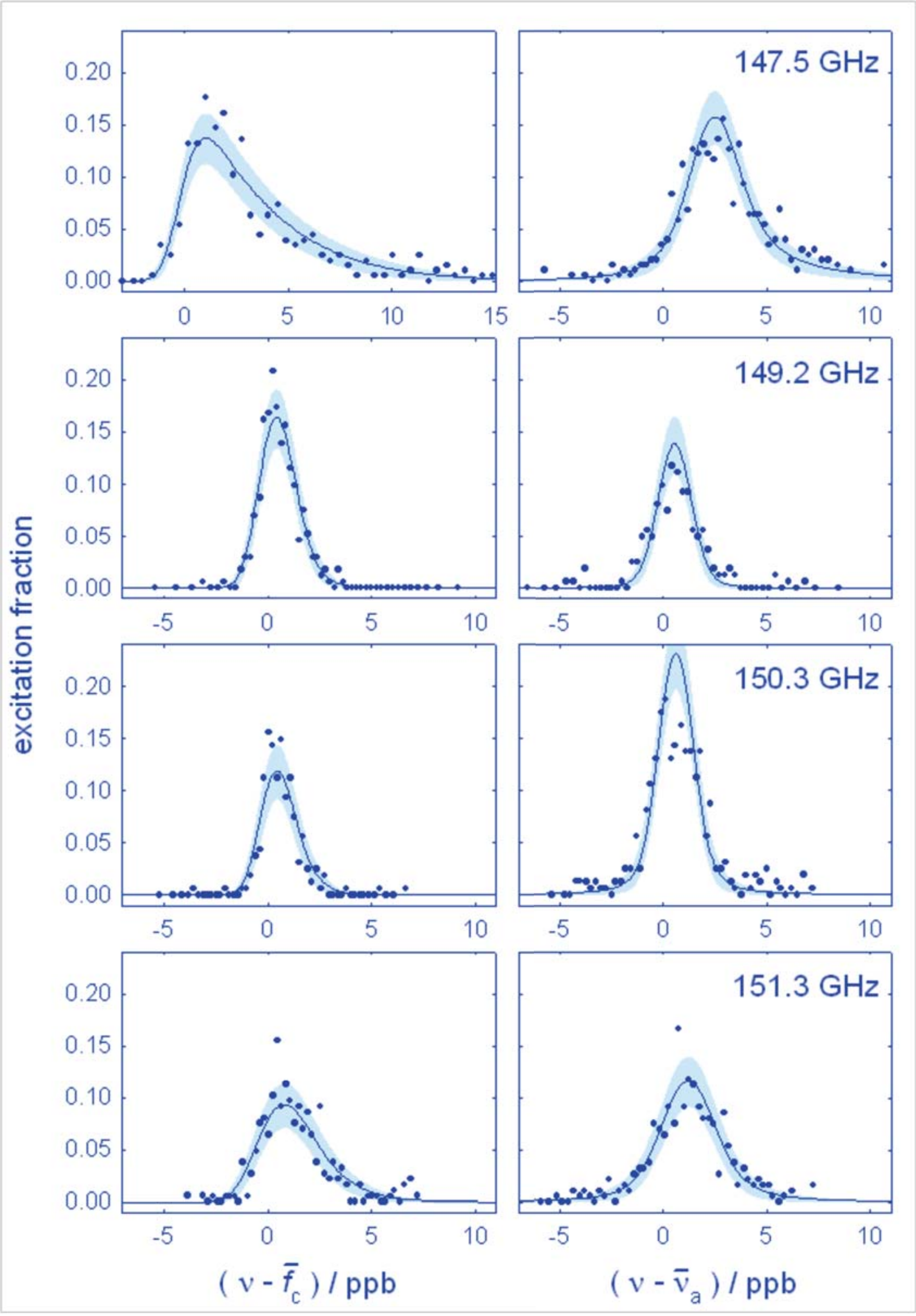
Lineshape Model Uncertainty

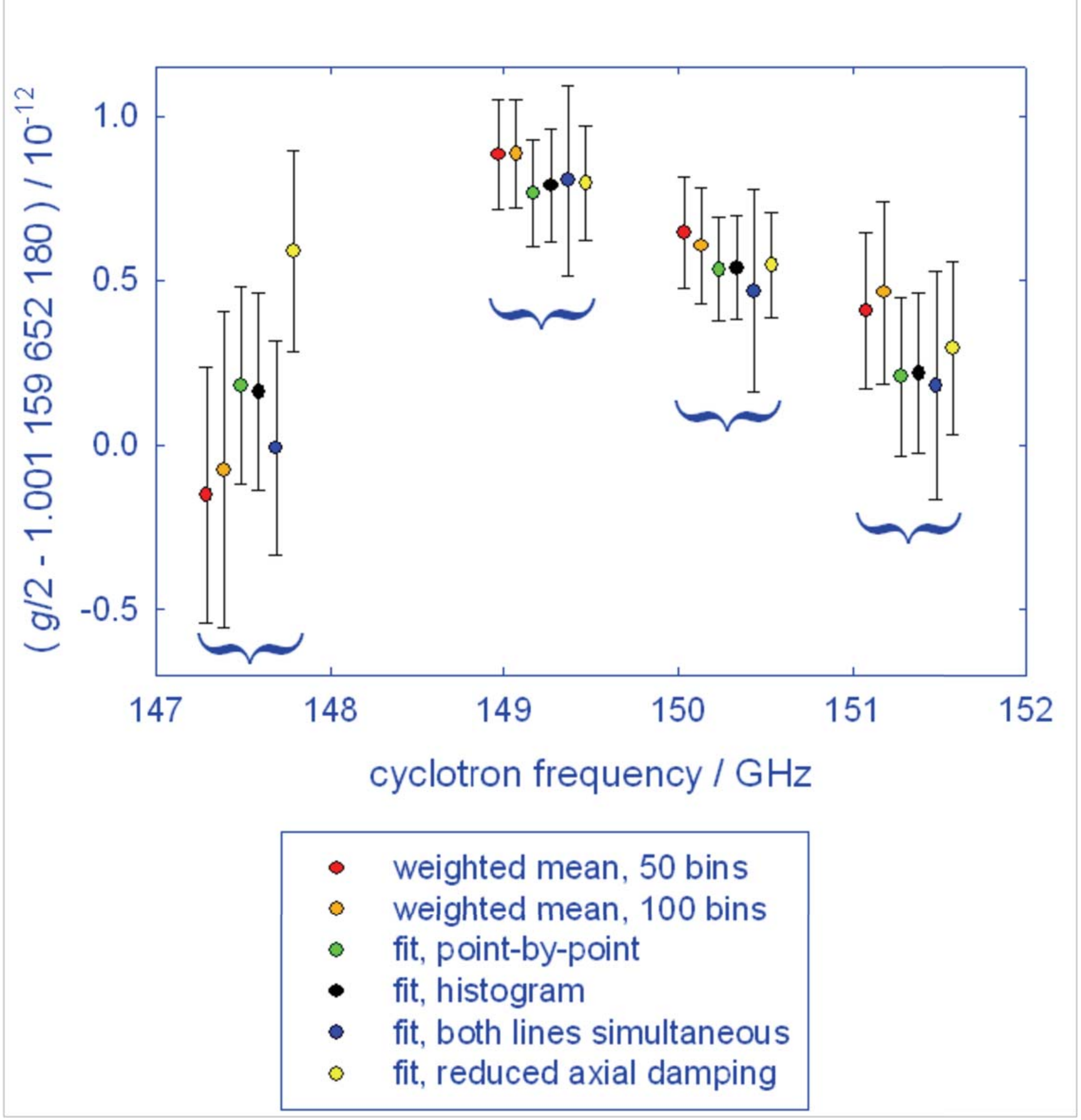


- Concern about magnetic field noise
- Get $\overline{f}_c$, $\overline{\nu}_a$ with a weighted-mean method
- Check with fits
- How well do they agree?

L.S. Brown, *Ann. Phys. (N.Y.)* **159**, 62 (1985)

Lineshape Model Uncertainty



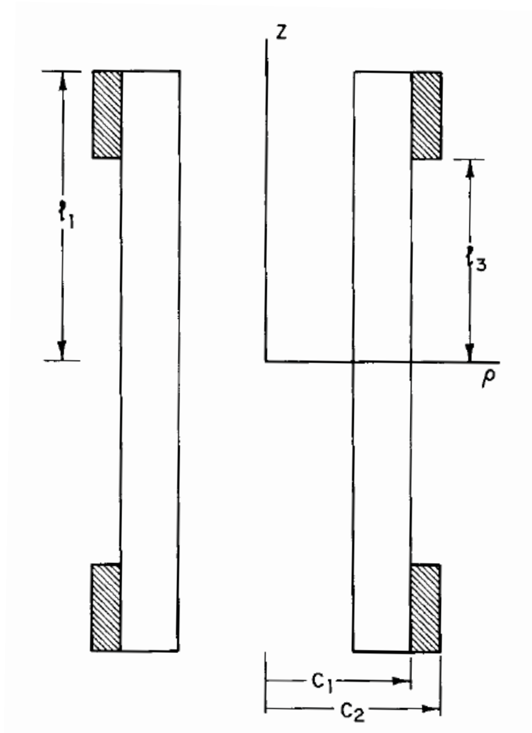


Outline

I. Introduction
II. Measurement overview
III. Novel techniques
IV. Measurement details
V. Uncertainties
VI. Experimental challenges
VII. What's next

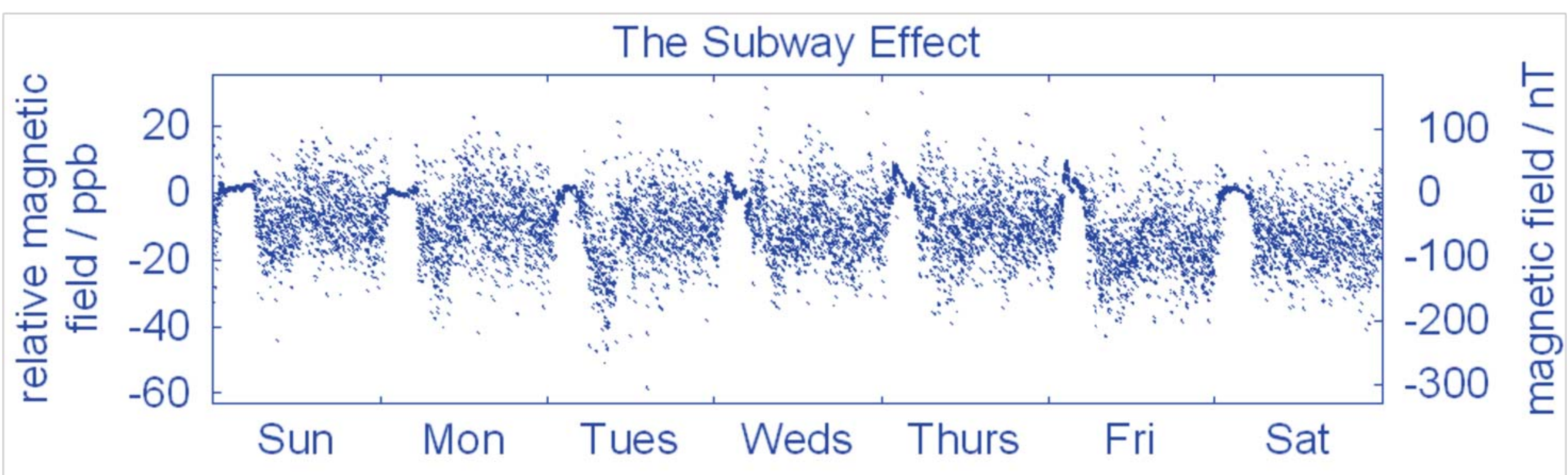
Magnetic Field Stability

Self-shielding solenoid reduces fluctuations by > 150.

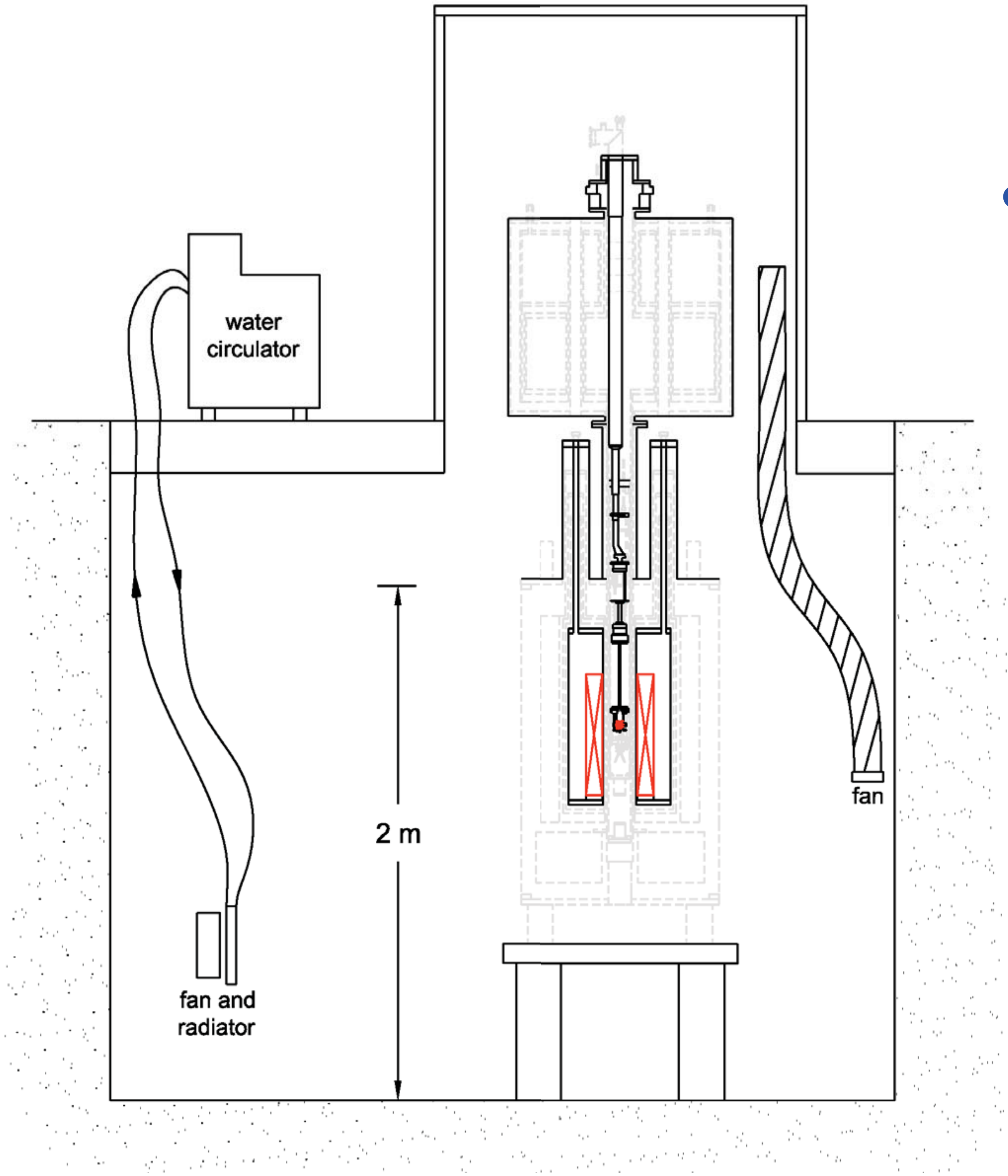


G. Gabrielse and J. Tan, *J. Appl. Phys*. **63**, 5143 (1988)

- External fluctuations
- Magnet settling

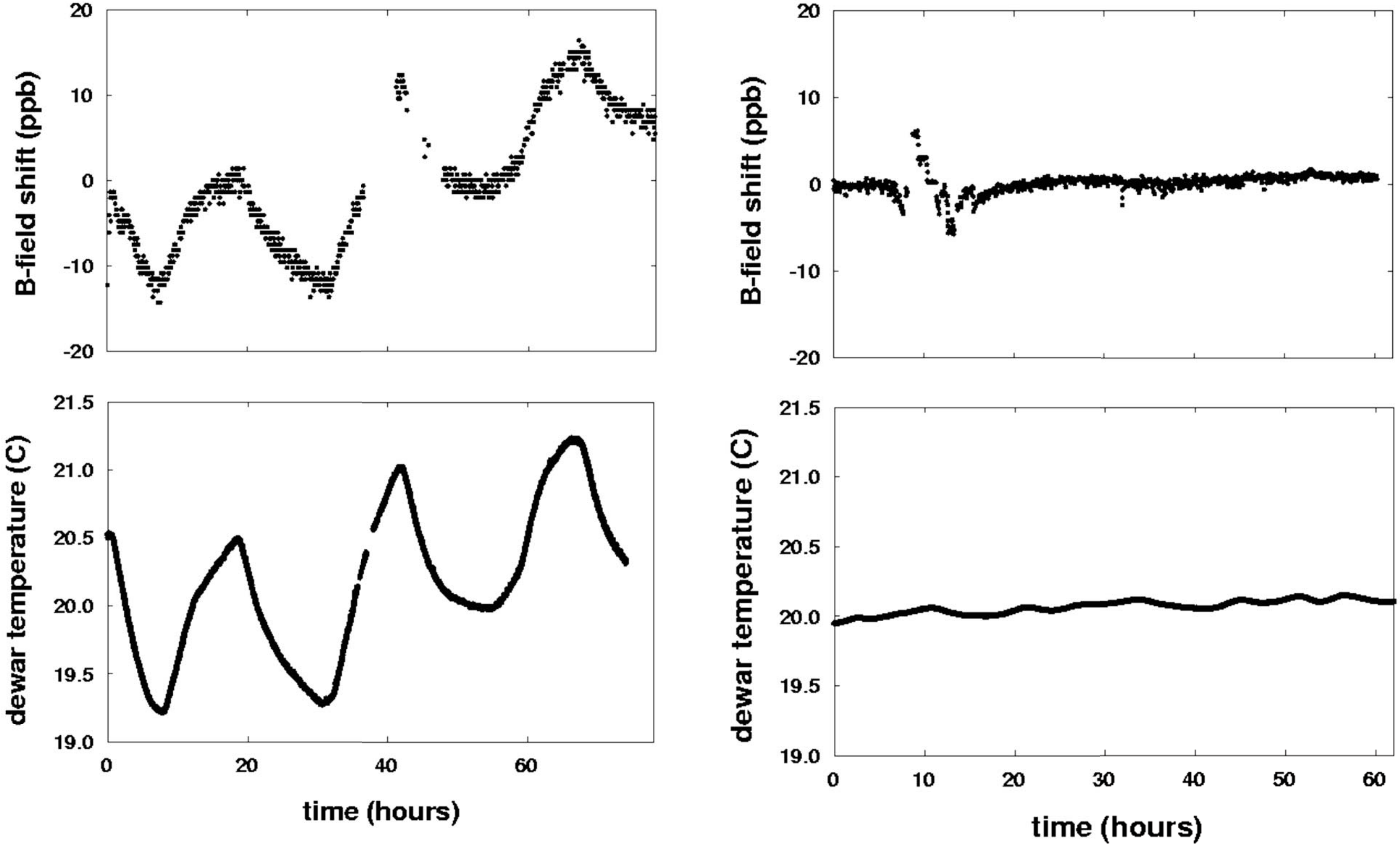


Magnetic Field Stability

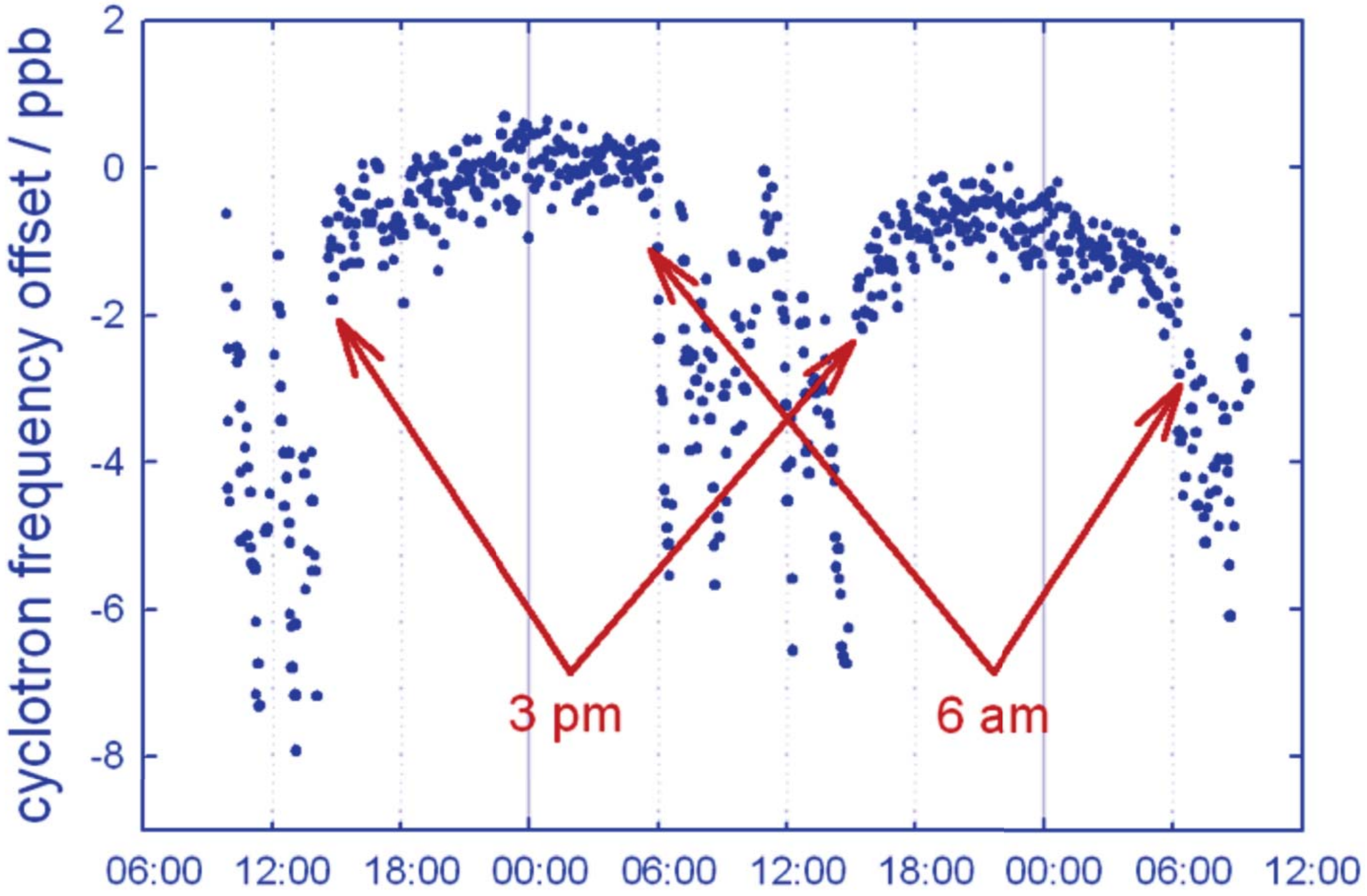


- Electrode motion in an inhomogeneous field

Temperature Regulation



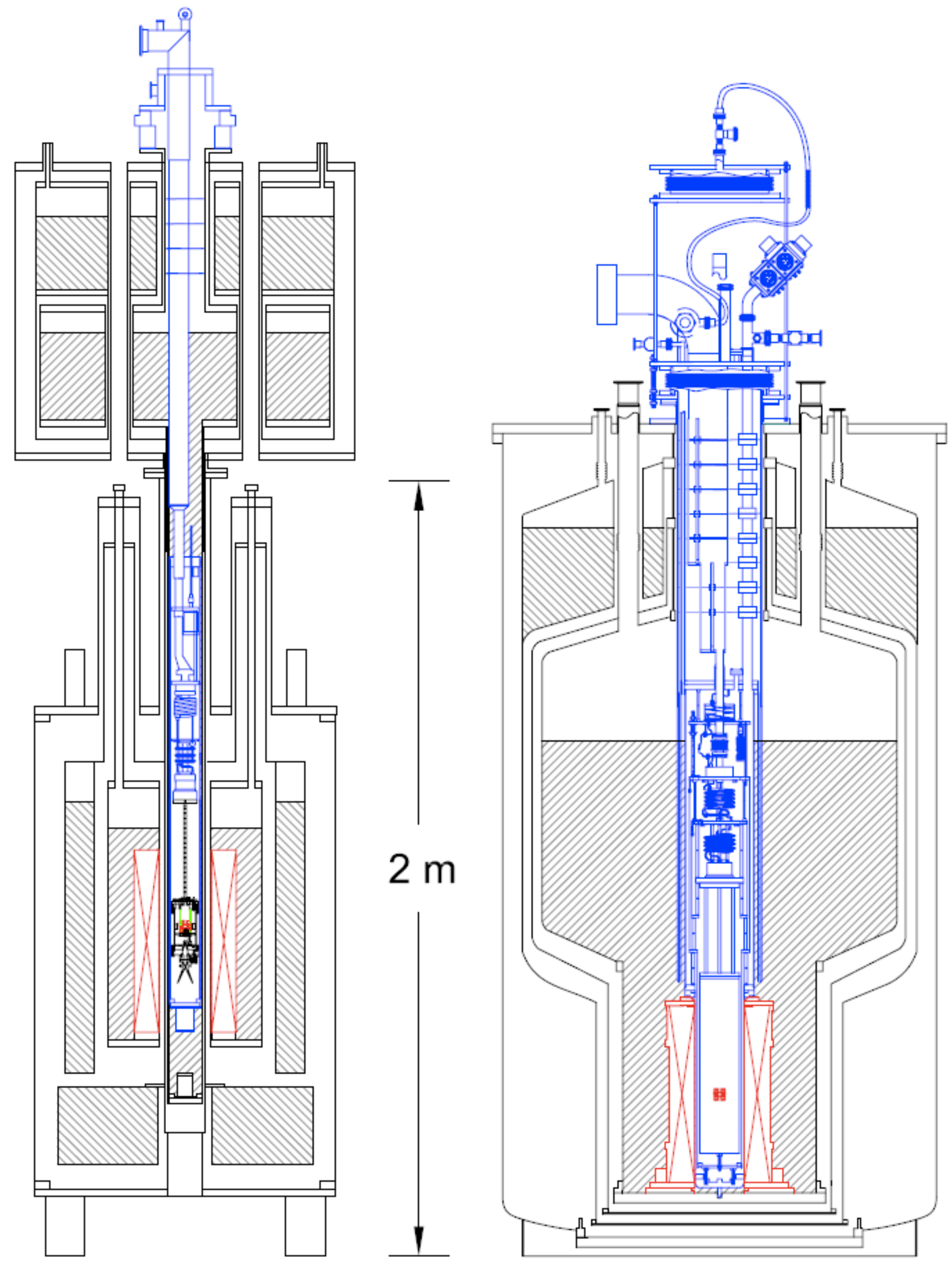
Construction



Outline

Short Term Outlook

New high-stability apparatus



Medium Term Outlook

- ➡ e^+ *g*-value / CPT test
- Cavity-assisted axial-cyclotron sideband cooling and other line-narrowing techniques
- Speed up the measurement cycle (π-pulses and adiabatic fast passage)
- ➡ μ_p, $\mu_{\bar{p}}$ (first single-proton spin flip, improve the antiproton measurement by 10^6)

Nick Guise

Josh Dorr

Shannon Fogwell

Long Term Outlook

- m_p / m_e
- Planar electrode geometries
- Electron as qubit

S. Stahl, *et al*, *Eur. Phys. J. D* **32**, 139 (2007)

Josh Goldman

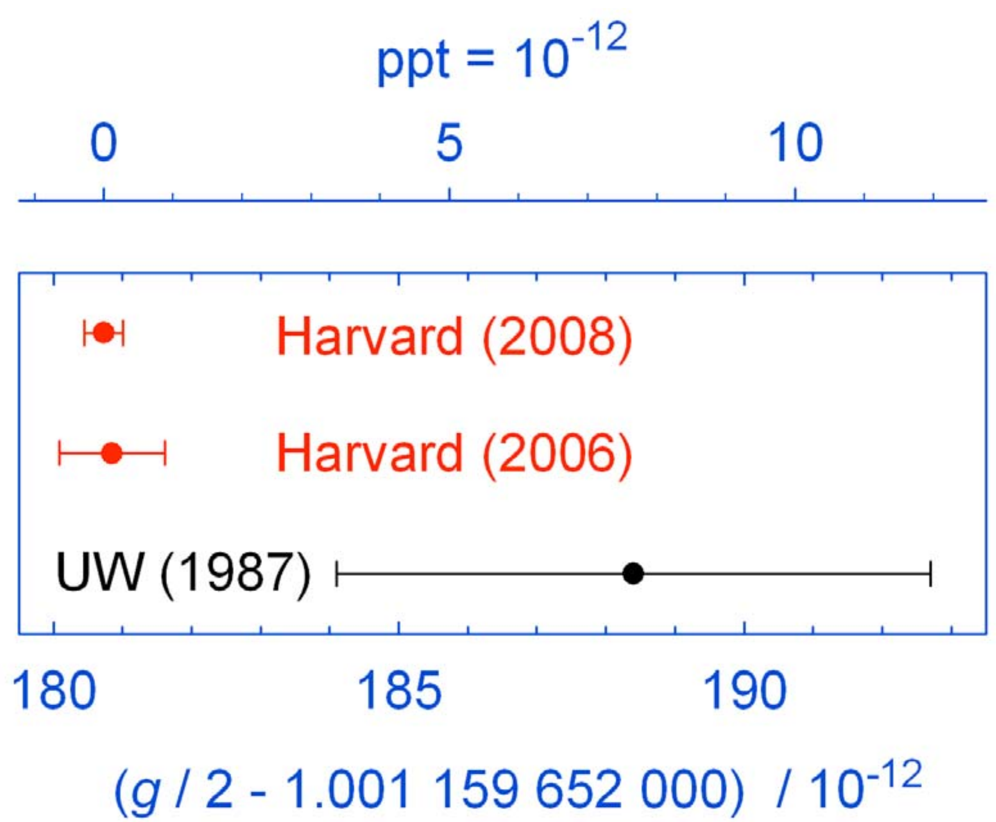
Jack DiSciacca

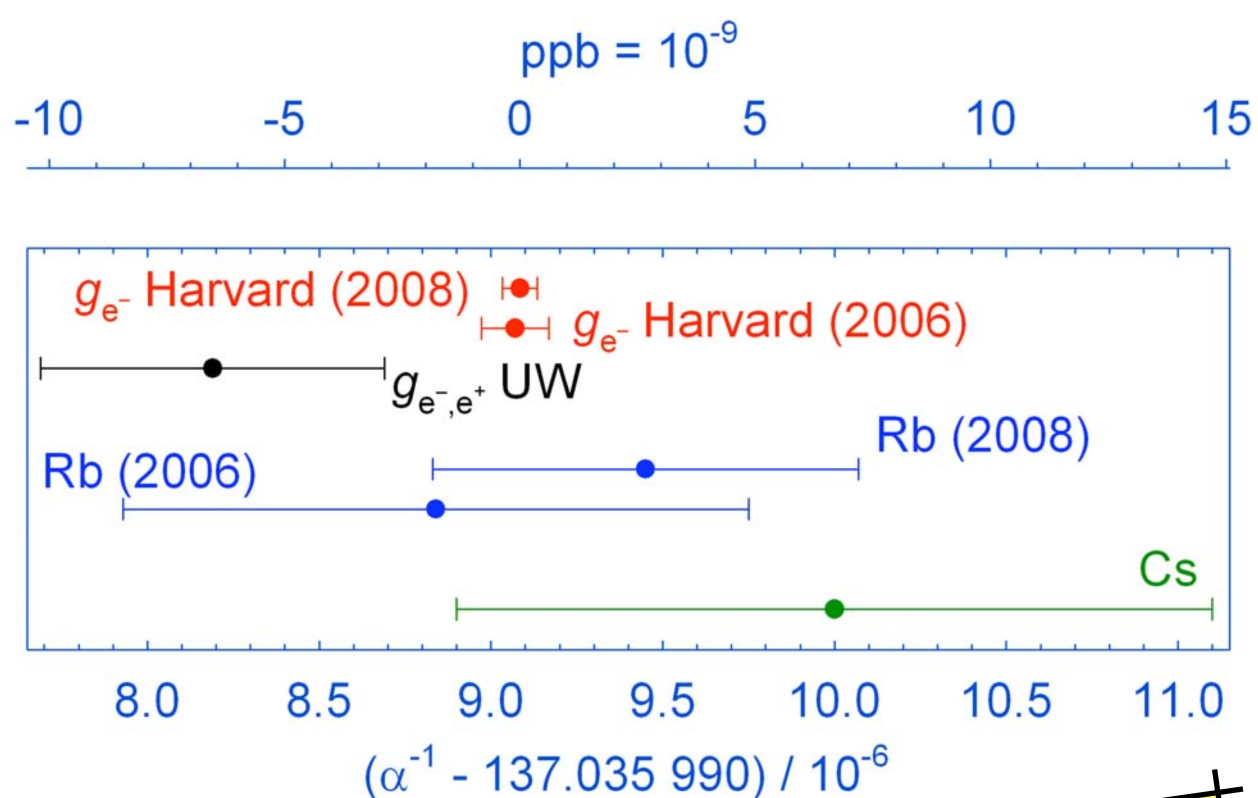
Summary

- First improvements in *g* in 20 years (factor of 15 total)
- Cavity effects no longer dominate the uncertainties
- More to come...

$g/2$ = 1.001 159 652 180 73 (28) [0.28 ppt]

α^{-1} = 137.035 999 084 (51) [0.37 ppb]





D. Hanneke, S. Fogwell, and G. Gabrielse, *Phys. Rev. Lett.* **100**, 120801 (2008)
B. Odom, D. Hanneke, B. D'Urso, and G. Gabrielse, *Phys. Rev. Lett.* **97**, 030801 (2006)
G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, and B. Odom, *Phys. Rev. Lett.* **97**, 030802 (2006). *Ibid*. **99** 039902(E) (2007)

High-Energy Physics with Low-Energy Symmetry Studies
Friday, May 14, 12:30pm, Miller Room