Measuring the Electron Magnetic Moment

Cavity Control in a Single-Electron Quantum Cyclotron

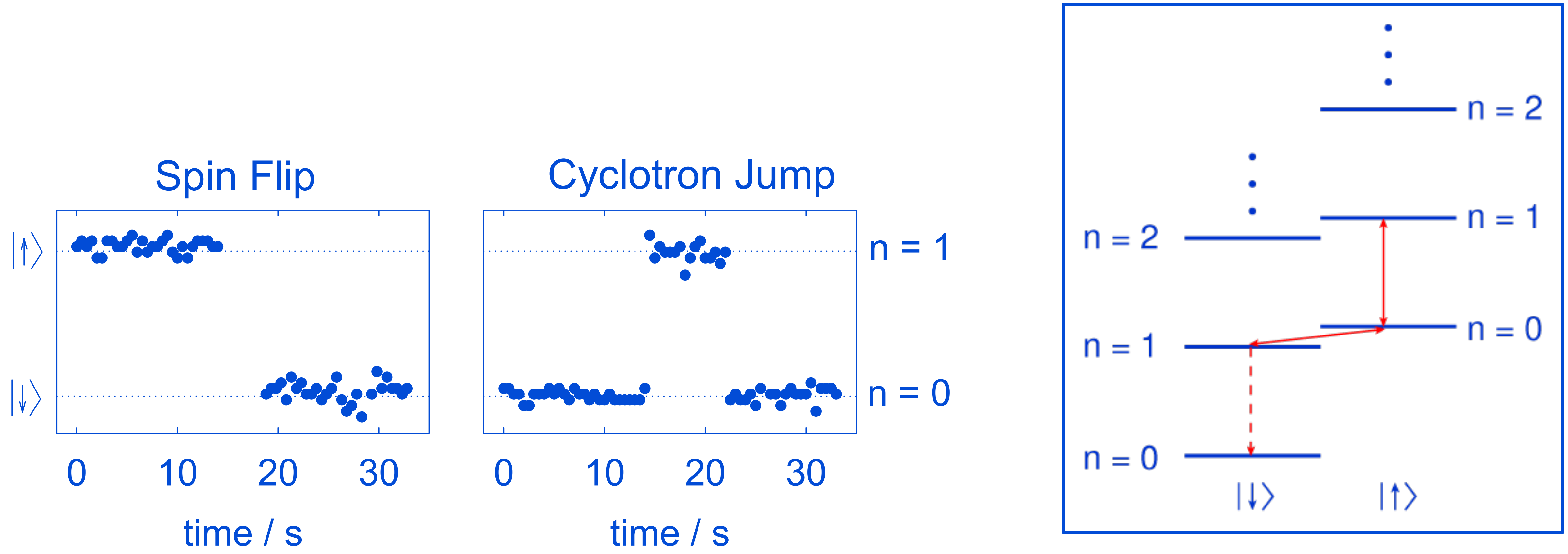
David Hanneke

Lepton Moments International Symposium

19 July 2010

The Quantum Cyclotron

- Single electron
- Resolve lowest cyclotron and spin states via QND measurement



Acknowledgements

Principal Investigator

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Postdocs

Maarten Jansen

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Graduate Students

Josh Dorr

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David Hanneke (2007)

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Brian D'Urso (2003)

Steve Peil (1999)

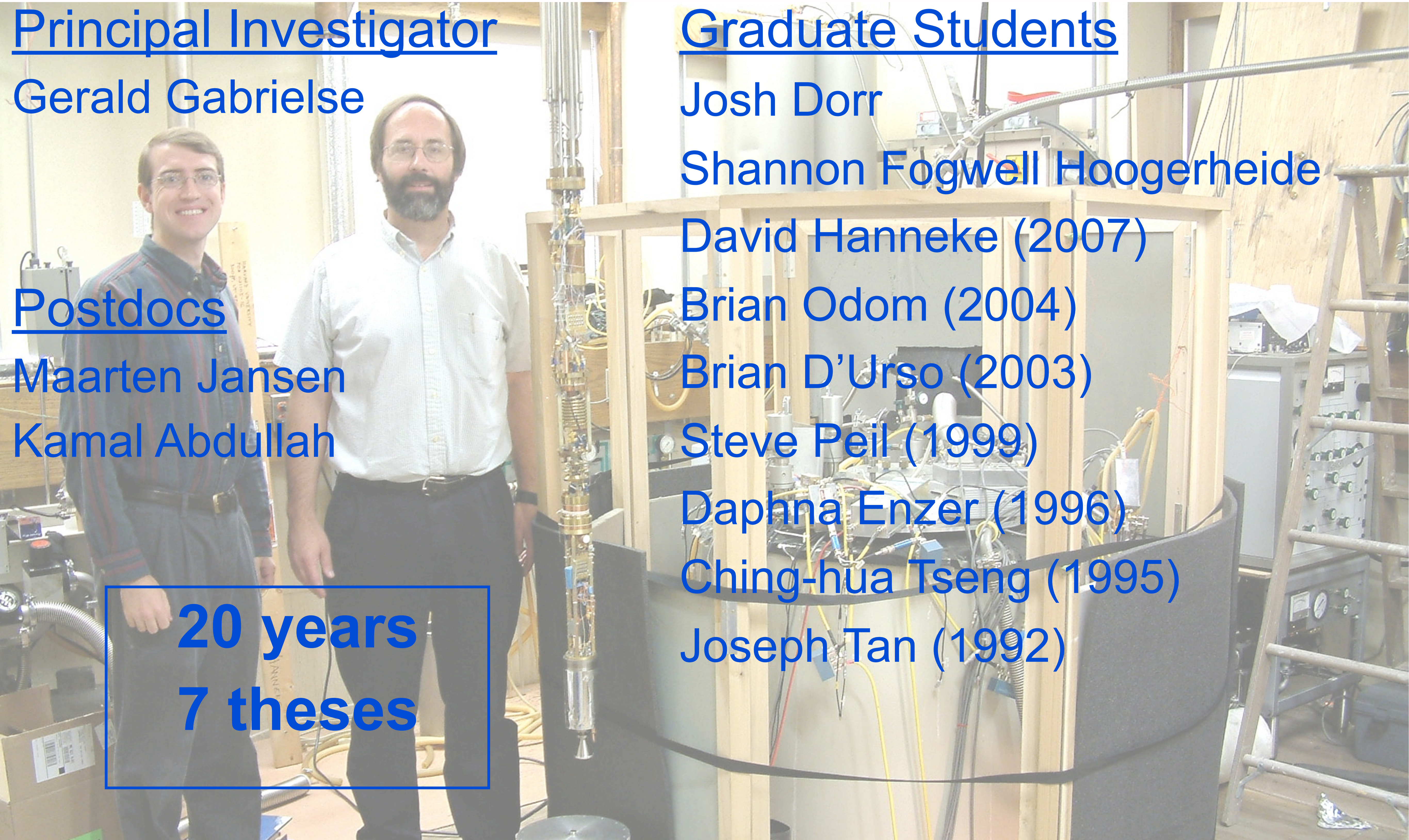
Daphna Enzer (1996)

Ching-hua Tseng (1995)

Joseph Tan (1992)

20 years

7 theses



Outline

I. Introduction
II. Measurement overview
III. Novel techniques
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V. Uncertainties
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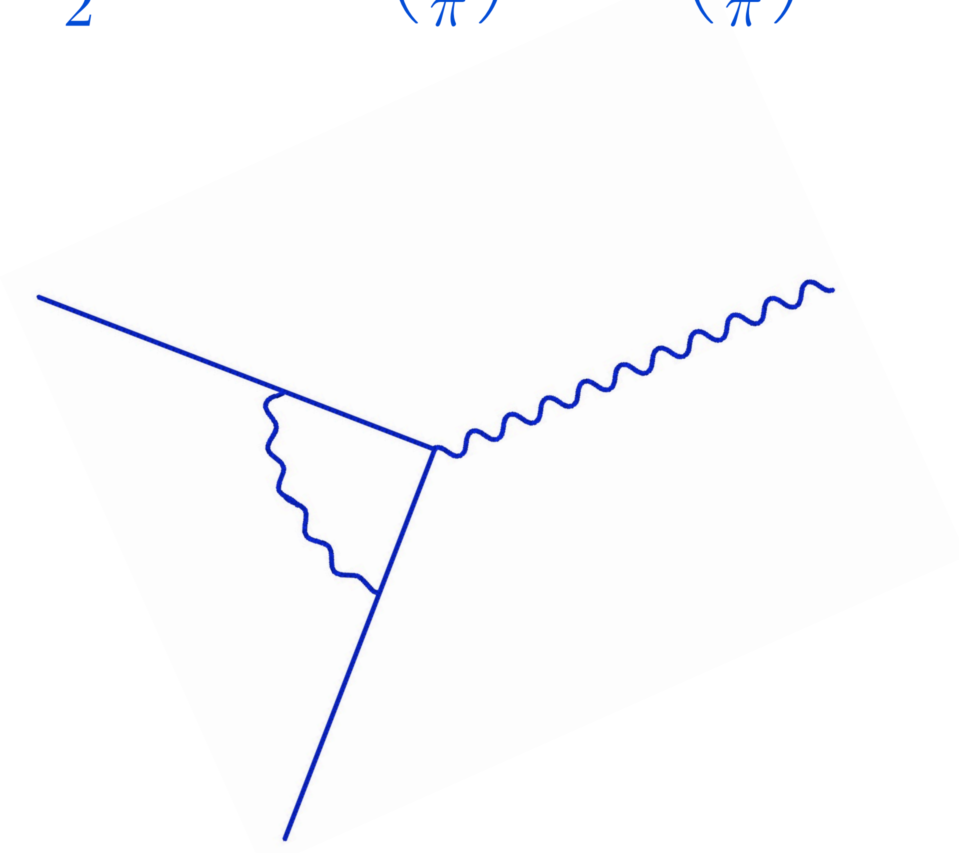
Magnetic Moments

$$\vec{\mu} = g \frac{-e\hbar}{2m} \frac{\vec{S}}{\hbar}$$

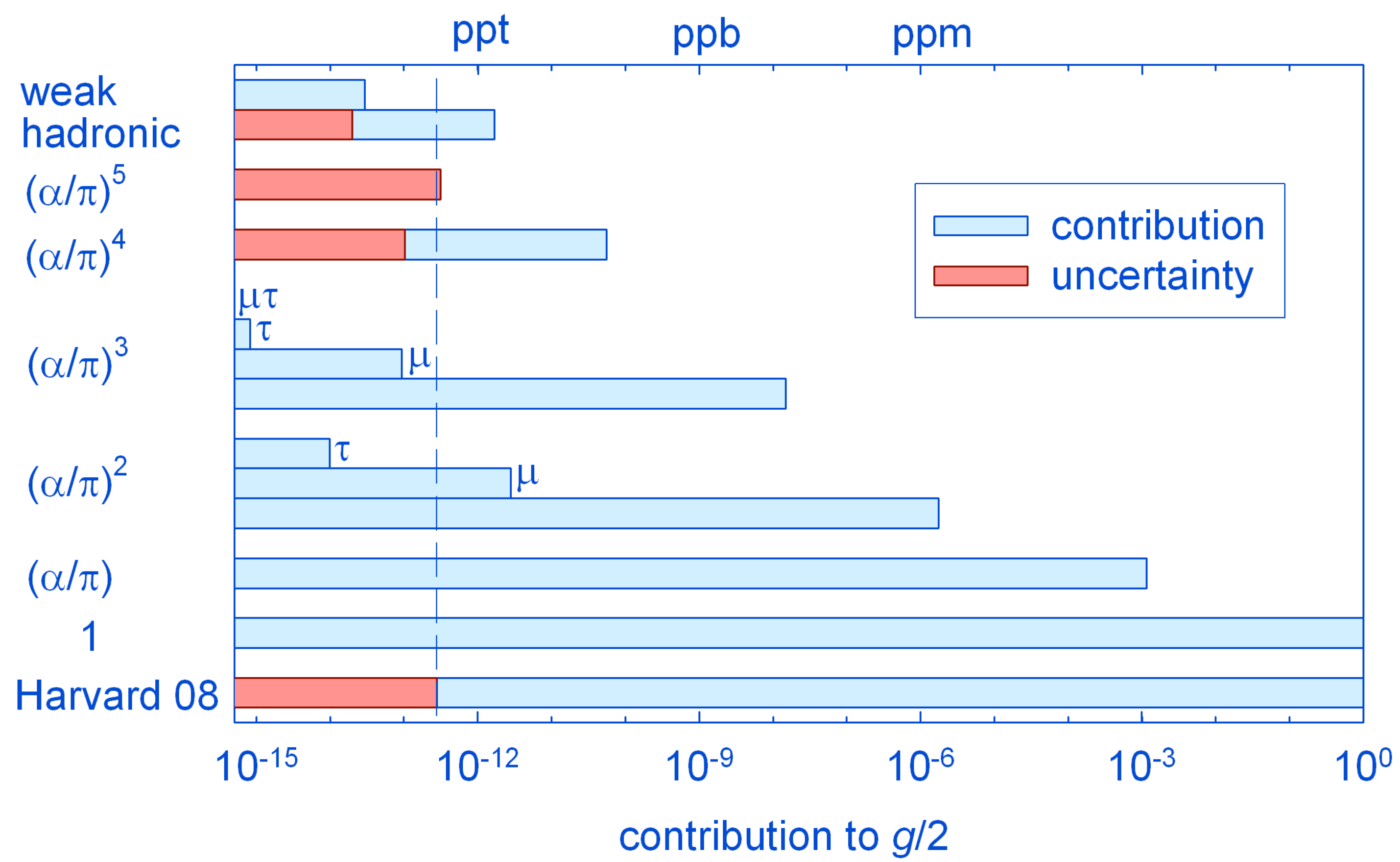
	Orbital angular momentum	Intrinsic angular momentum (Dirac point particle)	Structure of the vacuum (QED)	Structure of the particle (proton)
g =	1	2	2.002 319 304 …	5.585 …

α, *g*, and QED

$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + C_{10}\left(\frac{\alpha}{\pi}\right)^5 + \ldots + a^{\mu,\tau} + a^{\text{had}} + a^{\text{weak}}$$



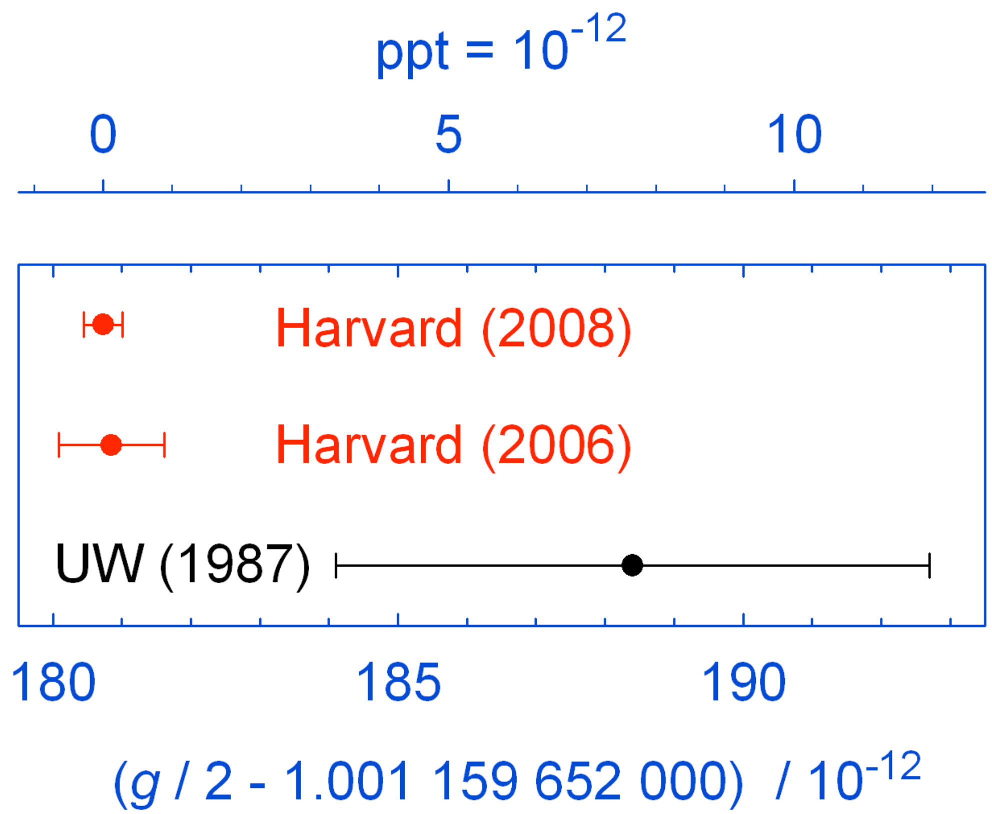
$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$$

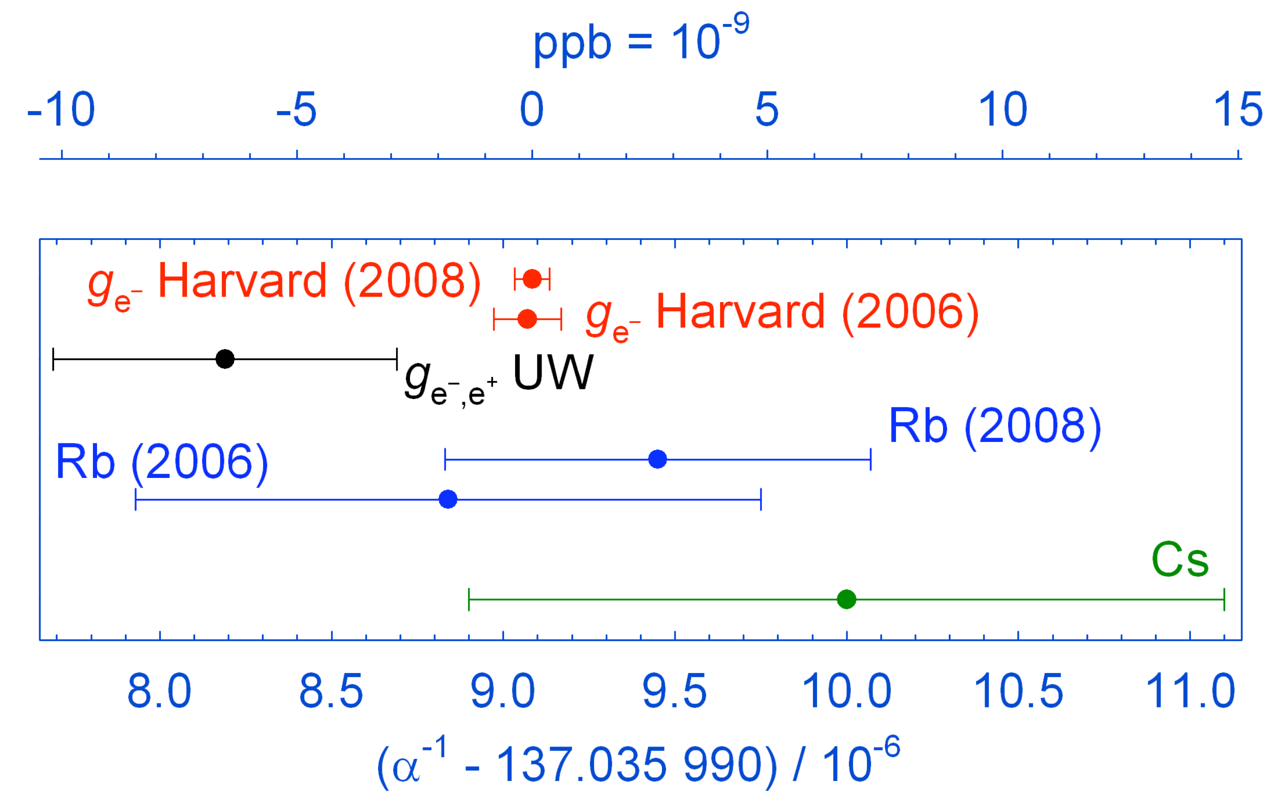


Measurement Results

$g/2$ = 1.001 159 652 180 73 (28) [0.28 ppt]

α^{-1} = 137.035 999 084 (51) [0.37 ppb]
= 137.035 999 084 (33) (39)
[0.24 ppb, exp.][0.28 ppb, th.]





D. Hanneke, S. Fogwell, and G. Gabrielse, *Phys. Rev. Lett.* **100**, 120801 (2008)

B. Odom, D. Hanneke, B. D'Urso, and G. Gabrielse,
Phys. Rev. Lett. **97**, 030801 (2006)

G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, and B. Odom,
Phys. Rev. Lett. **97**, 030802 (2006). *Ibid*. **99** 039902(E) (2007)

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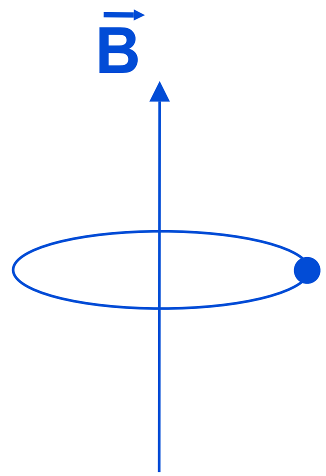
Experimenter's *g*

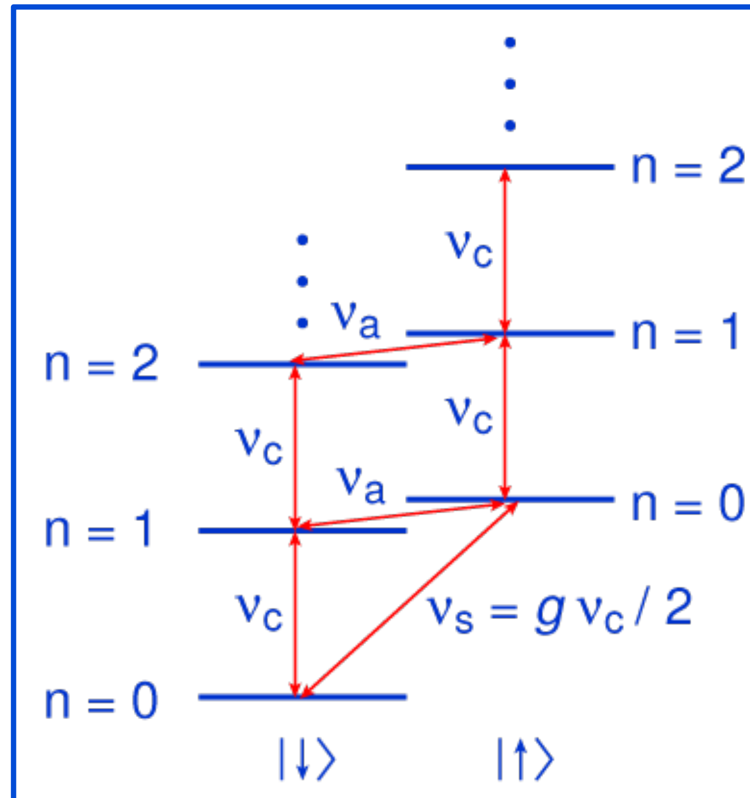
g in free-space (with a magnetic field)

What should we measure? Frequency!



$$\nu_c = \frac{eB}{2\pi m}$$

$$\nu_s = \frac{g}{2}\nu_c$$



$$\frac{g}{2} = \frac{\nu_s}{\nu_c} = 1 + \frac{\nu_s - \nu_c}{\nu_c} = 1 + \frac{\nu_a}{\nu_c}$$

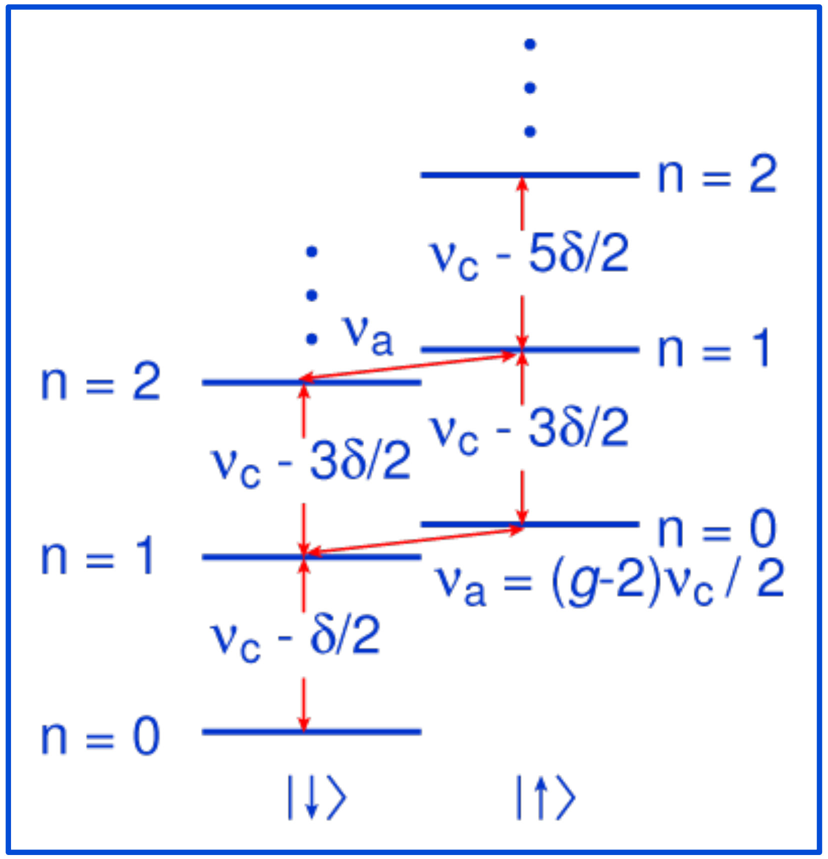
$$\frac{\nu_a}{\nu_c} = \frac{g-2}{2}$$

Experimenter's *g*

Special Relativistic corrections

$$\frac{\delta}{\nu_c} = \frac{h\nu_c}{mc^2} \approx 10^{-9}$$

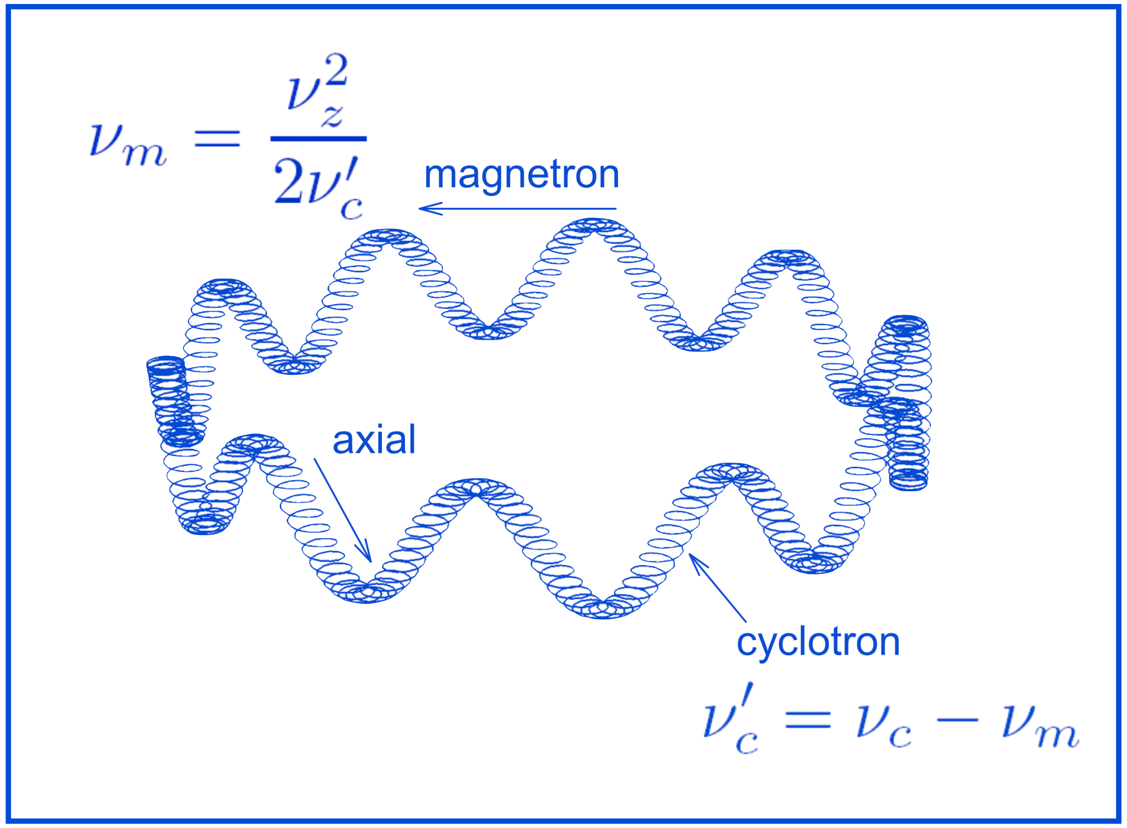
- The cyclotron is an anharmonic oscillator.
- No relativistic correction to ν_a.

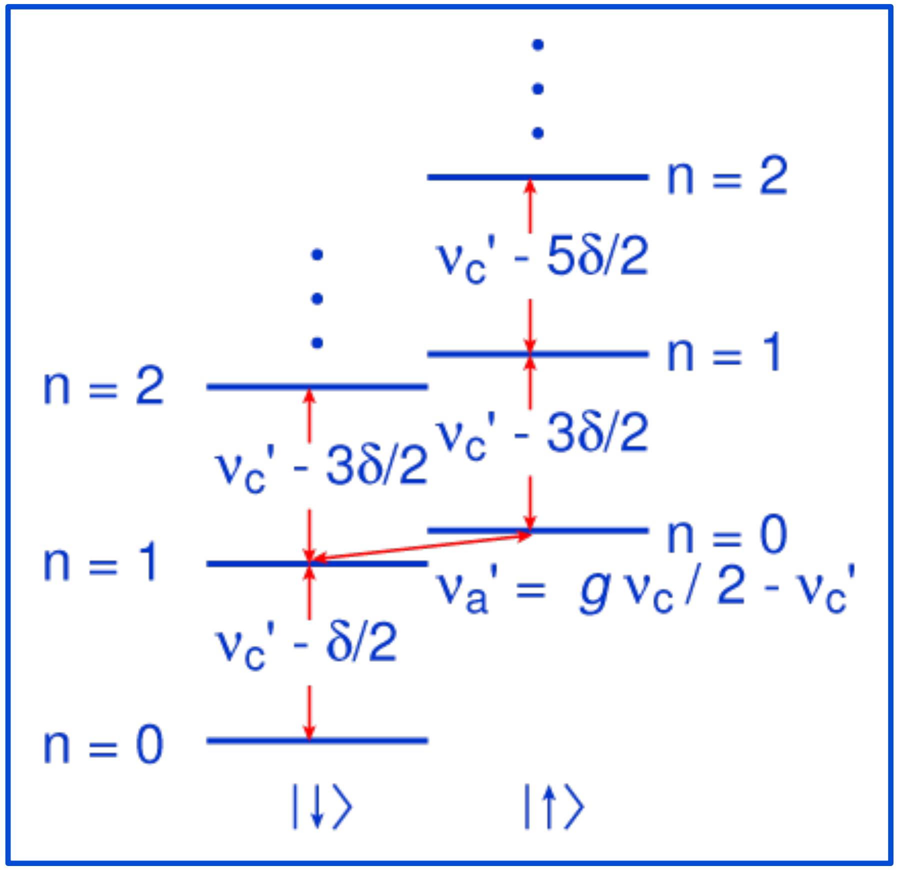


Experimenter's *g*

Penning Trap's electrostatic quadrupole

$$V \sim 2z^2 - x^2 - y^2$$





$$\nu_a' = \nu_s - \nu_c' = \frac{g}{2}\nu_c - \nu_c' = \frac{g-2}{2}\nu_c + \nu_m$$

Experimenter's *g*

A real Penning trap has

- distortions in the electrostatic quadrupole
- misalignment of the quadrupole axis and **B**

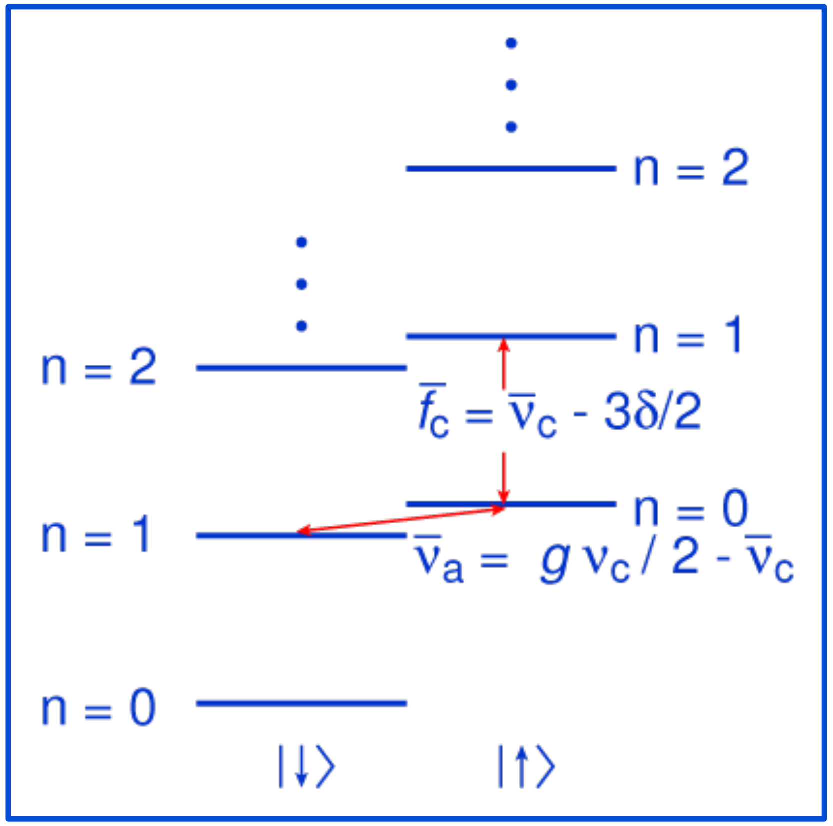
Brown-Gabrielse Invariance Theorem

$$\nu_c = \sqrt{(\bar{\nu}_c)^2 + (\bar{\nu}_z)^2 + (\bar{\nu}_m)^2}$$

L.S. Brown and G. Gabrielse, *Rev. Mod. Phys.* **58**, 233 (1986)

$$\frac{g}{2} \simeq 1 + \frac{\bar{\nu}_a - \dfrac{\bar{\nu}_z^2}{2\bar{f}_c}}{\bar{f}_c + 3\delta/2 + \dfrac{\bar{\nu}_z^2}{2\bar{f}_c}} + \frac{\Delta\omega}{\omega}$$

cavity shift



required hierarchy

$$\bar{\nu}_c \gg \bar{\nu}_z \gg \bar{\nu}_m \gg \delta$$

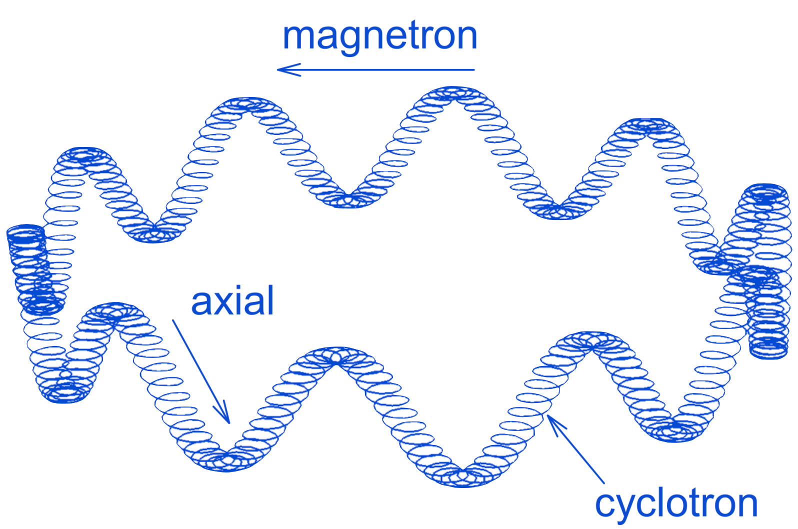
Our Trap Frequencies

B ~ 5.36 T

V_0 ~ 101.4 V

d ~ 3.5 mm (0.14 in)

motion	frequency	damping time
axial	200 MHz	0.2 s
cyclotron	150.0 GHz	5 s
spin	150.2 GHz	2 yr
magnetron	133 kHz	4 Gyr

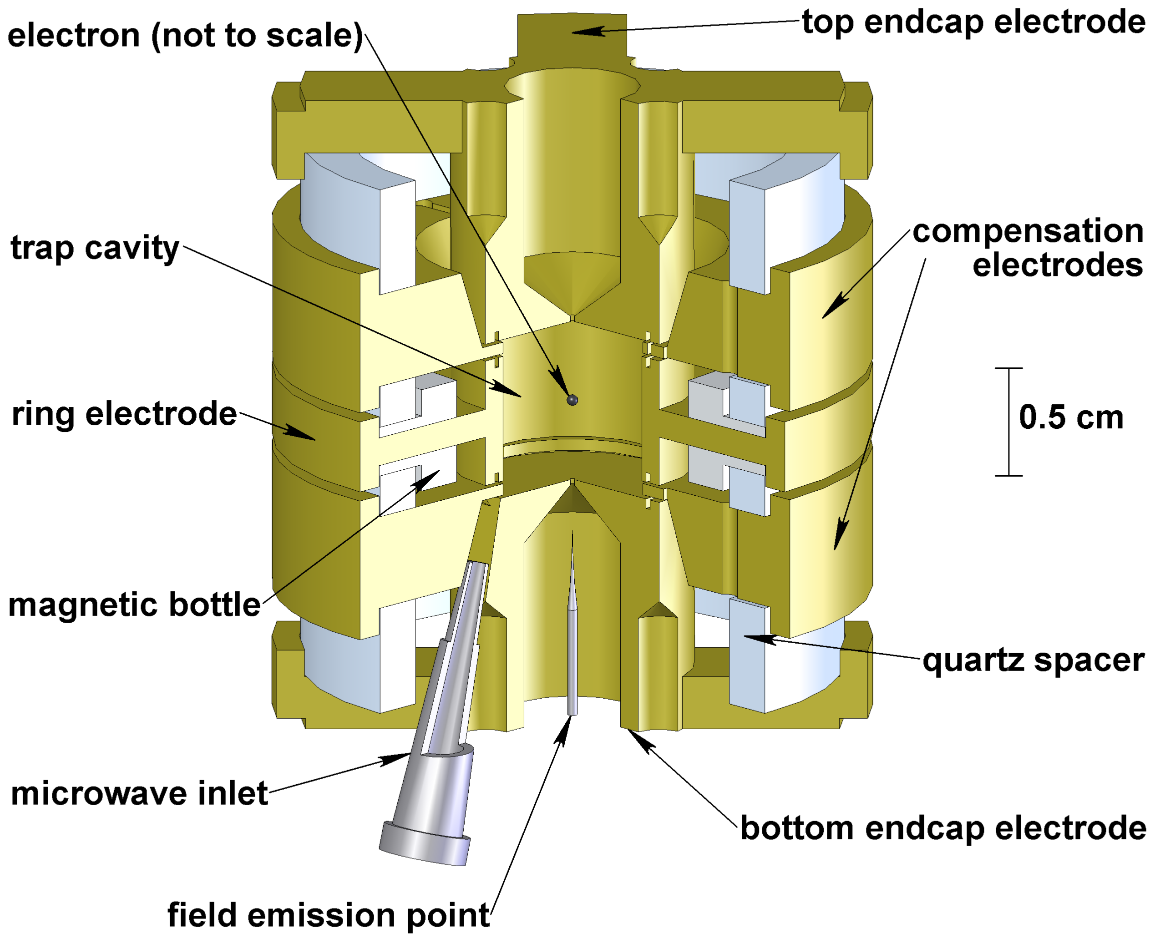


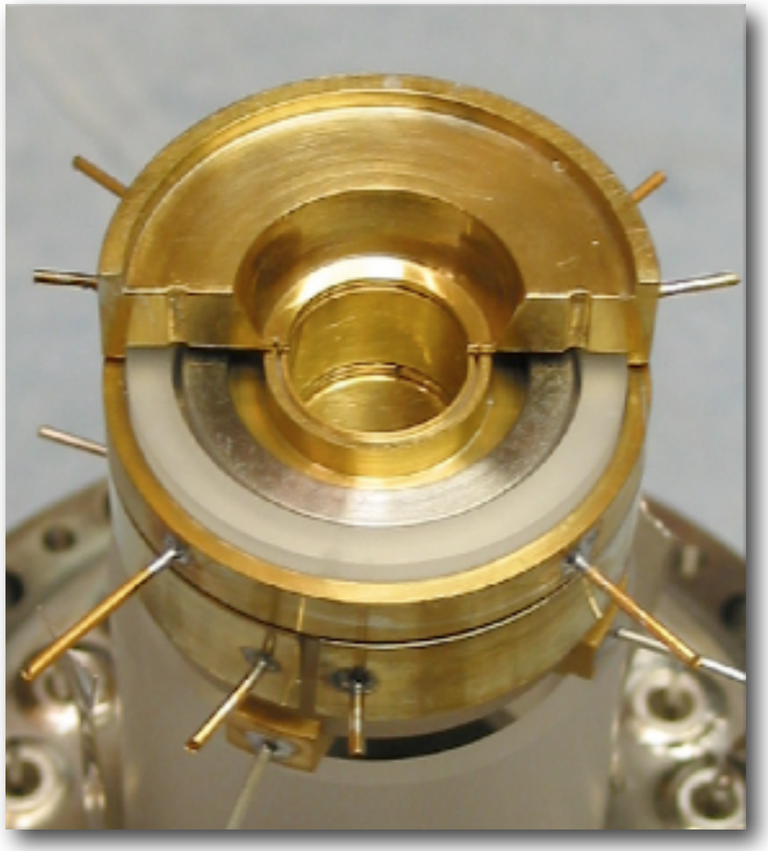
hierarchy satisfied

$$\bar{\nu}_c \gg \bar{\nu}_z \gg \bar{\nu}_m \gg \delta$$

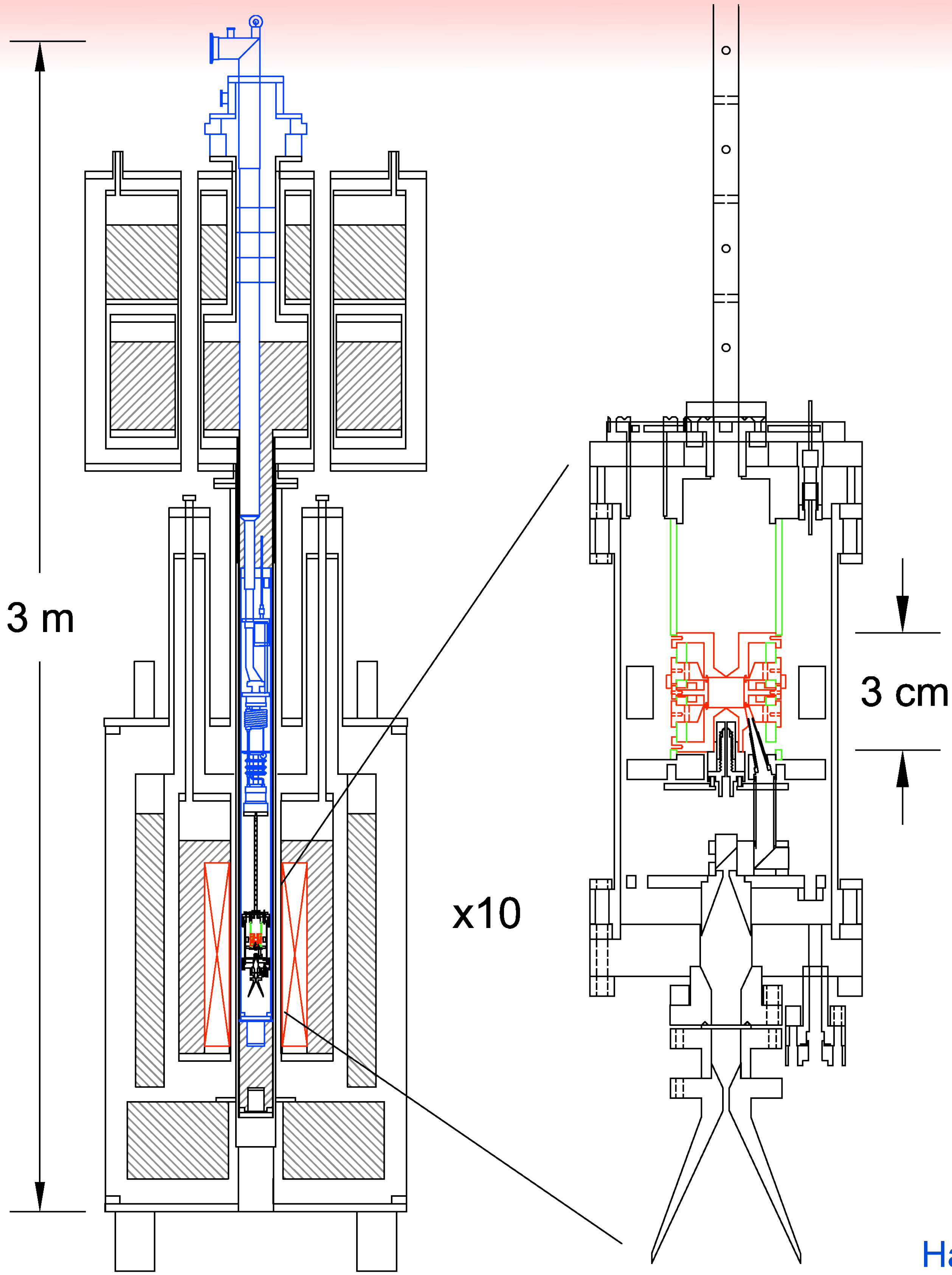
δ ~ 180 Hz

Our Trap

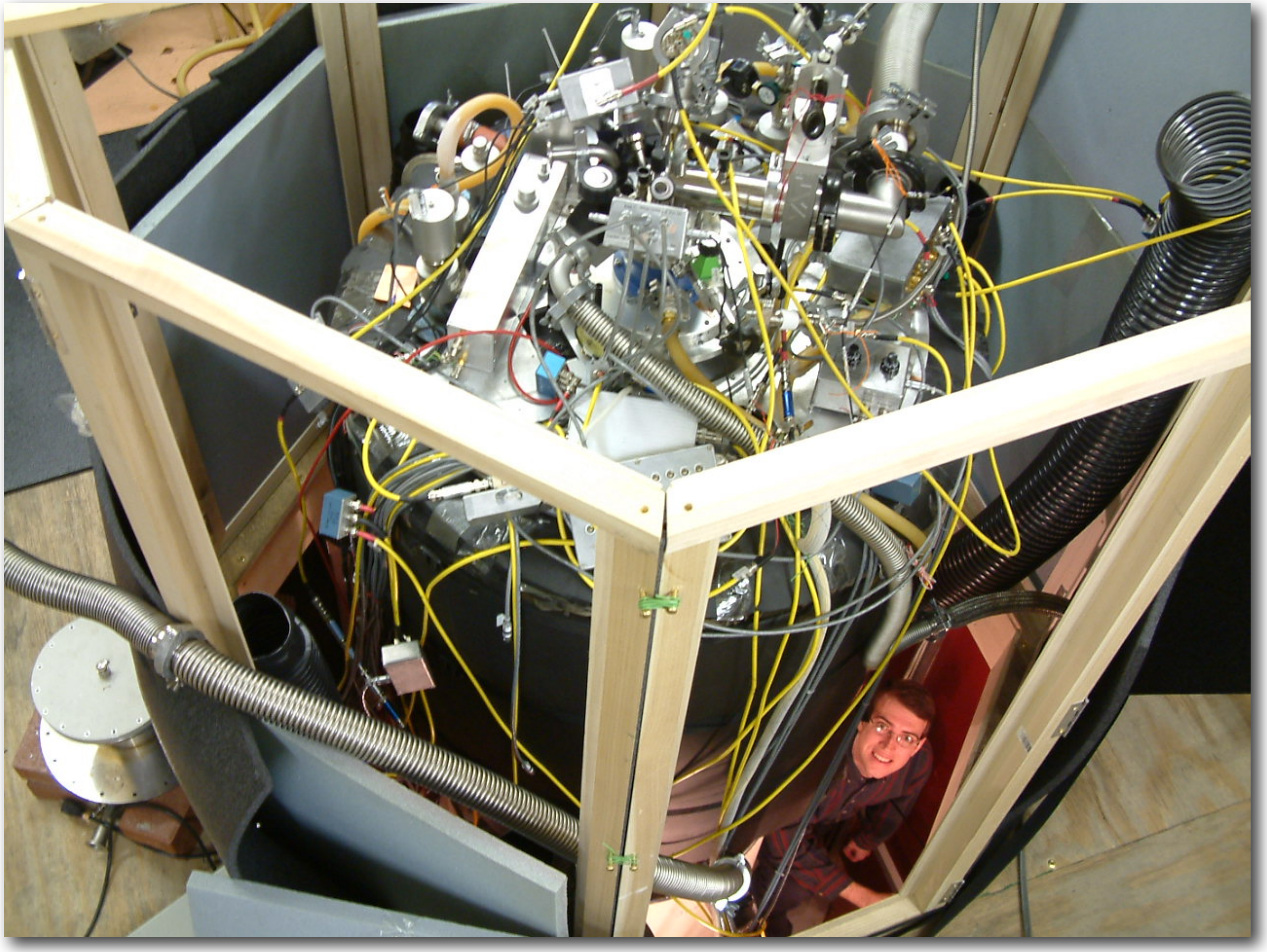

electron (not to scale)
top endcap electrode
trap cavity
compensation electrodes
ring electrode
0.5 cm
magnetic bottle
quartz spacer
microwave inlet
bottom endcap electrode
field emission point

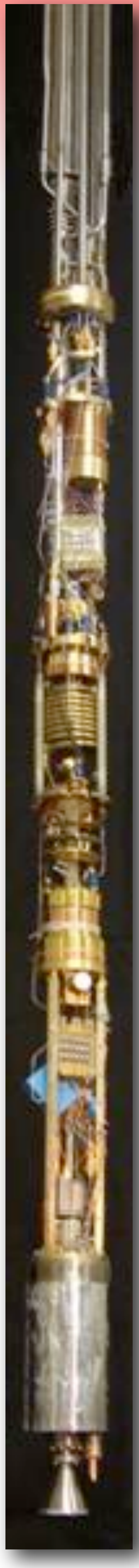


The Whole Apparatus



A Tabletop Experiment





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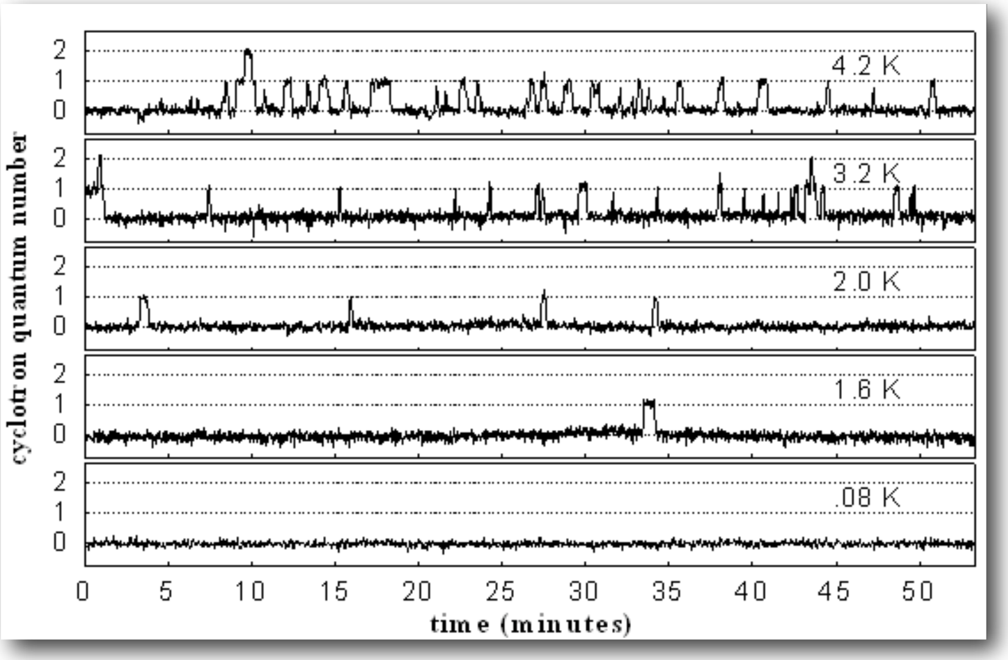
Low Temperatures

Cool the blackbody photons (⟨n⟩ << 1)

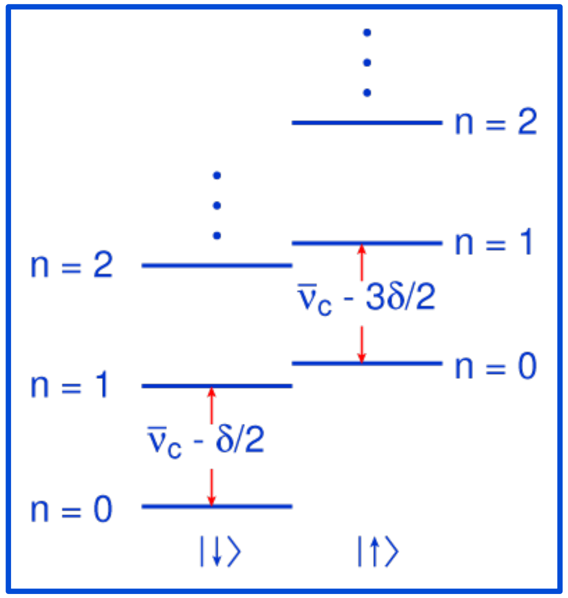
$$h\nu_c/k_B \approx 7.2\ \mathrm{K}$$

Quantum jumps

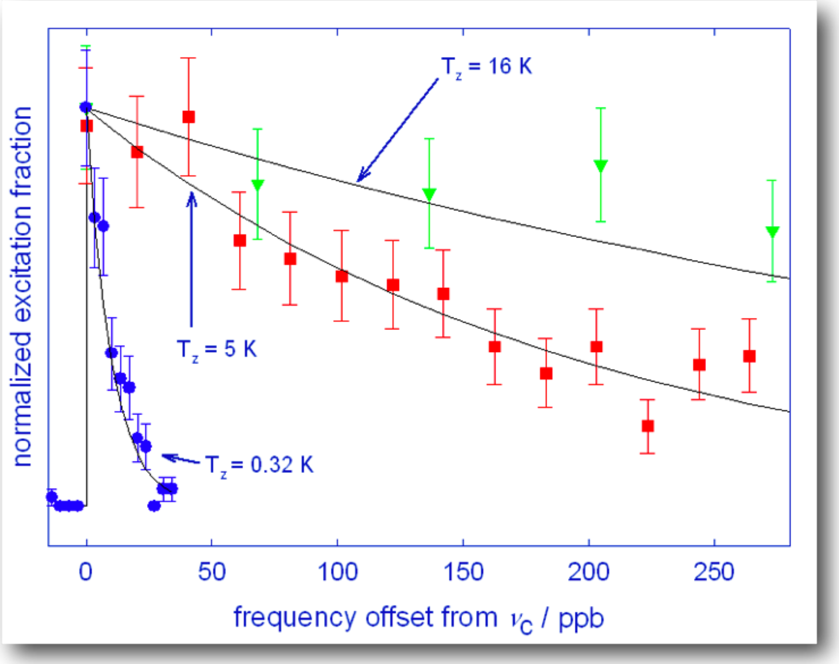


S.Peil and G.Gabrielse, *Phys. Rev. Lett.* **83**, 1287 (1999)

Exact relativistic shifts



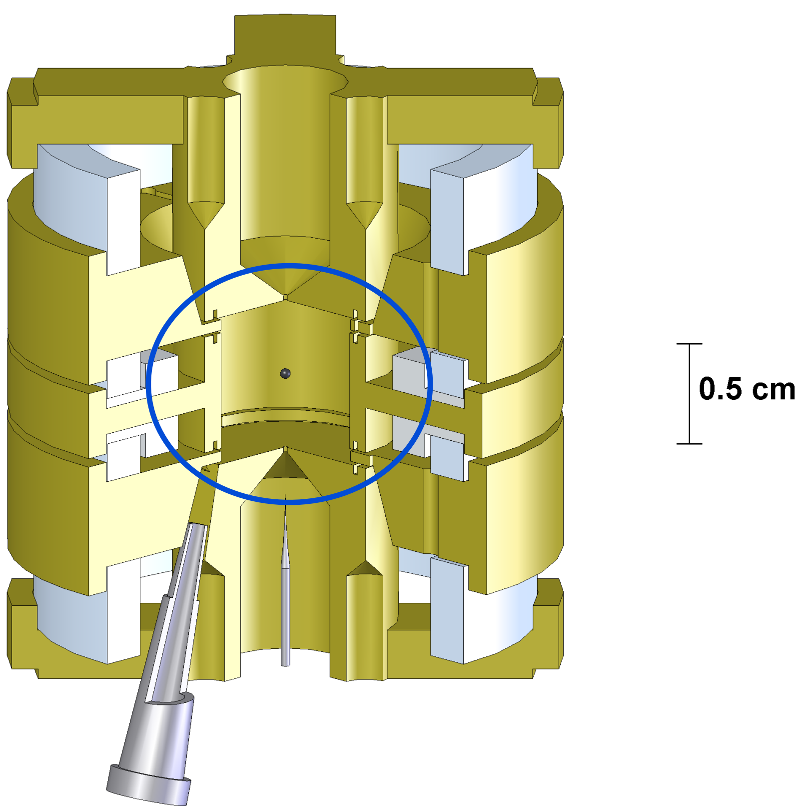
Narrow line widths

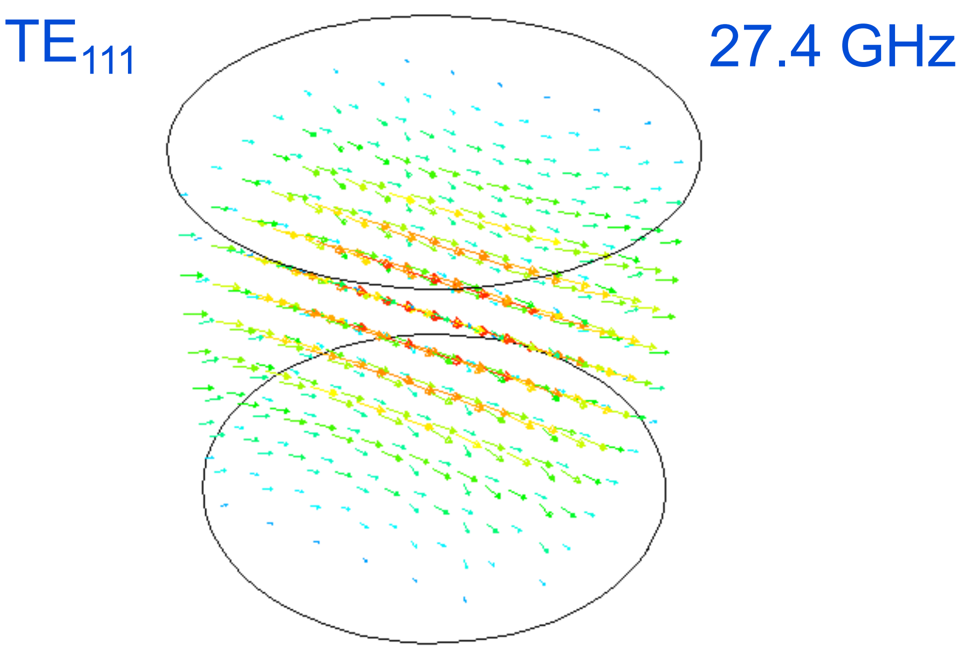


Cavity Control

The radiation modes of a cylindrical cavity are

- Well understood
- Resonant near ν_c = 150 GHz (λ ~ 2 mm)
- Coupled to the cyclotron motion if the geometry is right



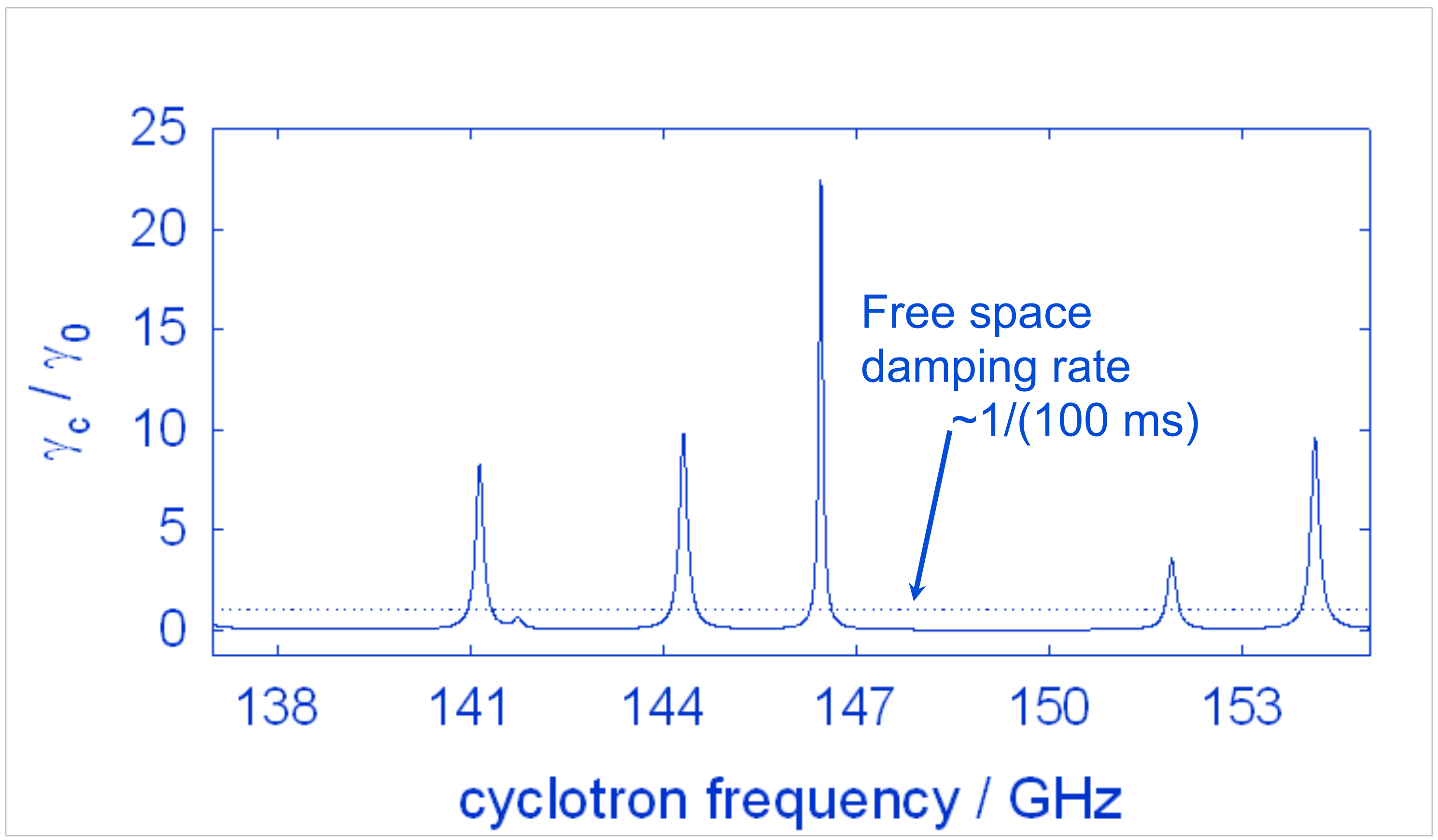


J.D. Jackson, *Classical Electrodynamics*, 3rd Ed., Sect. 8.7

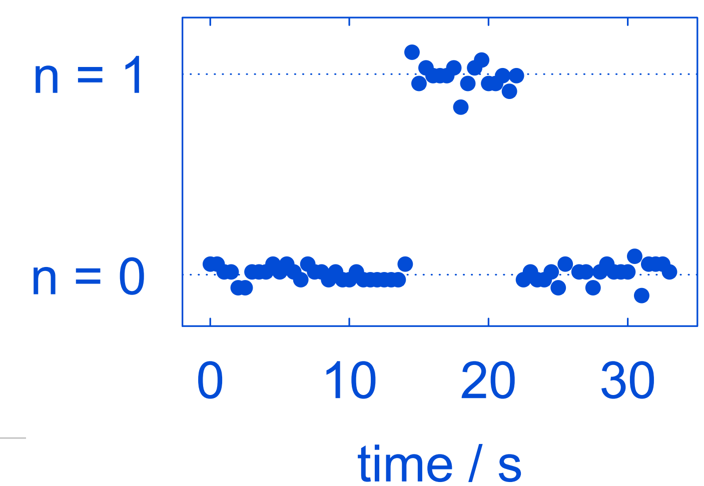
L.S. Brown *et al.*, *Phys. Rev. A* **32**, 3204 (1985)

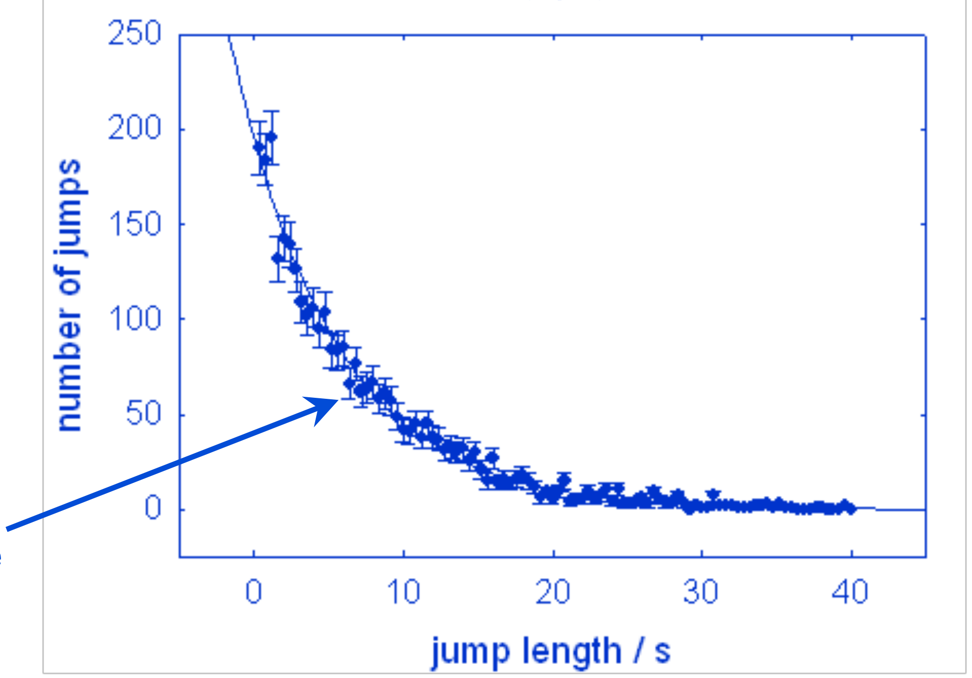
Cavity Control

Control the cyclotron damping rate, γ_c



Inhibited spontaneous emission

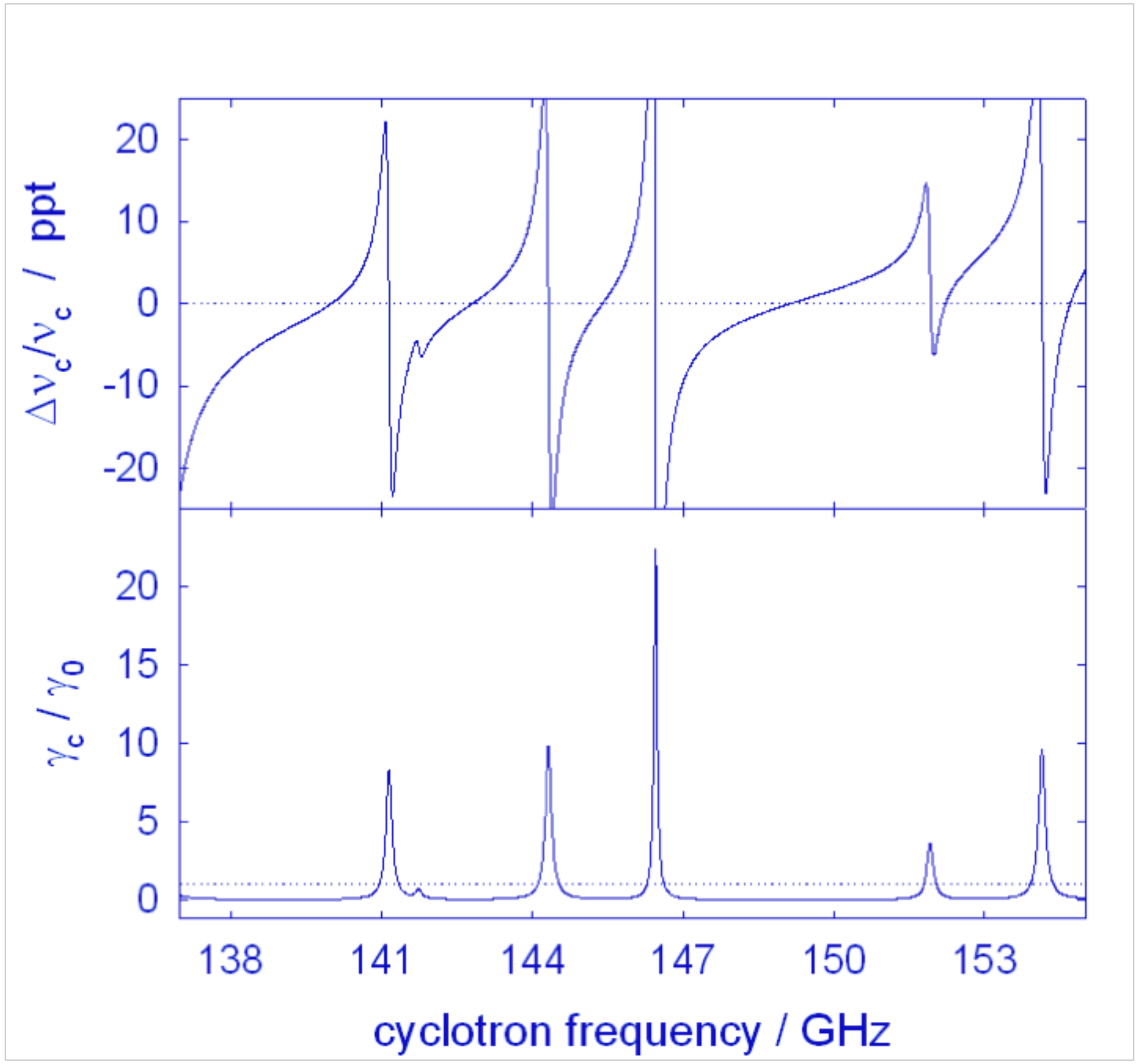




$\tau = (6.70 \pm 0.18)$ s $= 73.6 \times \tau_{\text{free space}}$

Cavity Control

Shift the cyclotron frequency



$$\frac{\Delta g}{g} = -\frac{\Delta \nu_c}{\nu_c}$$

Details later…

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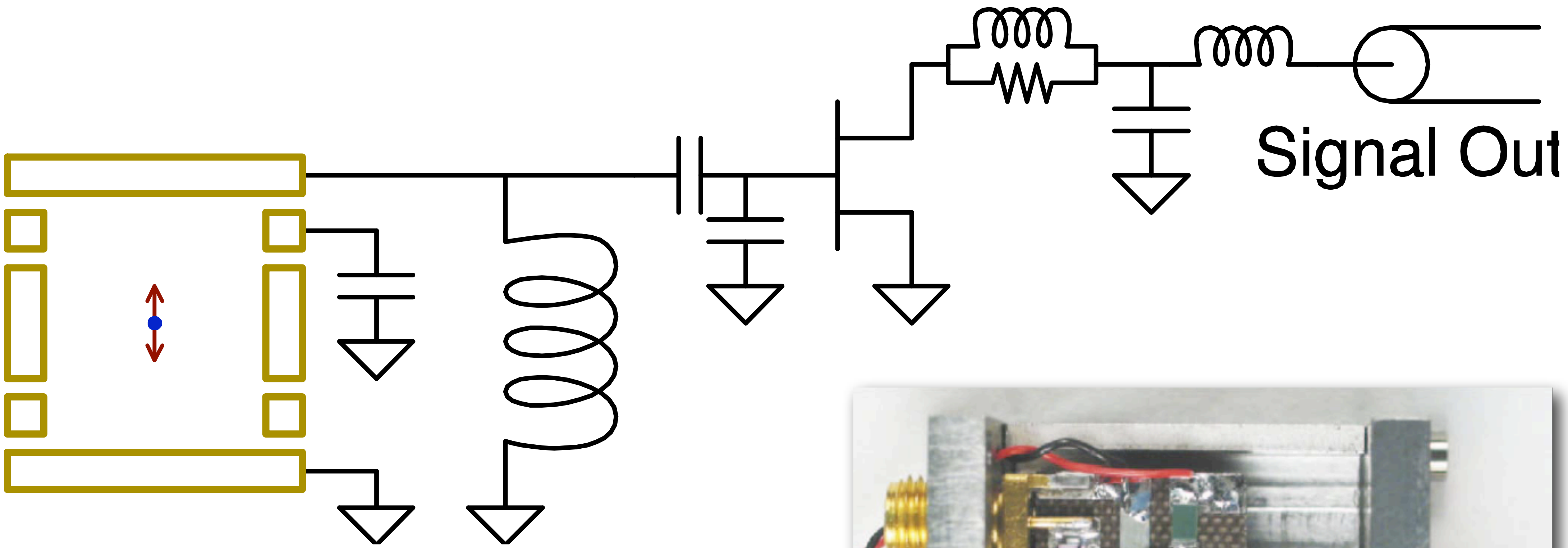
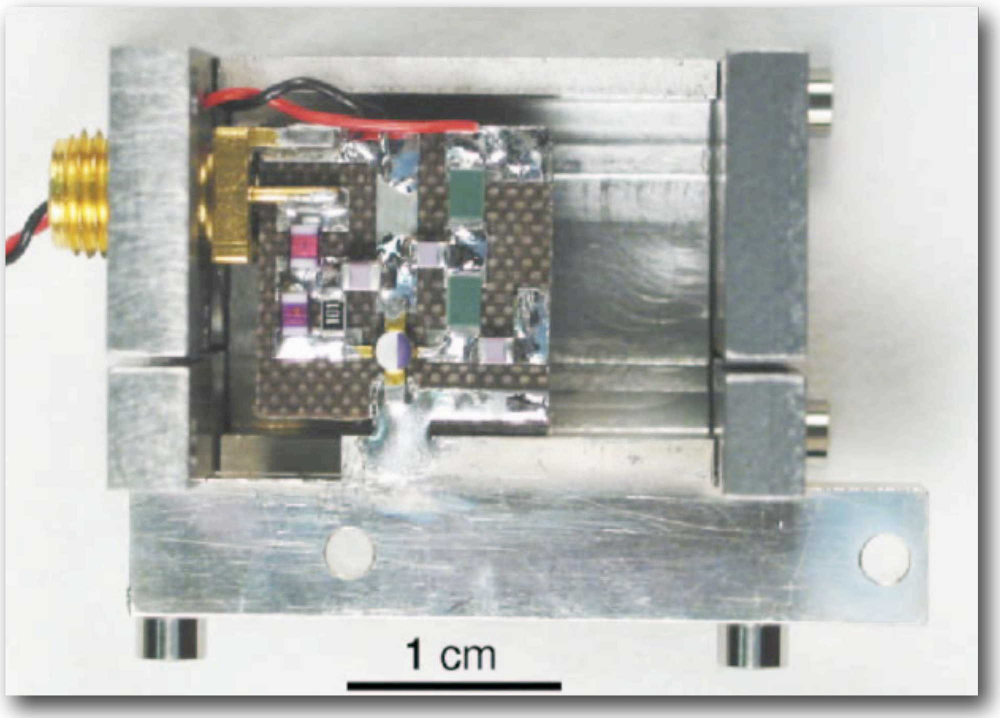
Determining *g*

$$\frac{g}{2} \simeq 1 + \frac{\bar{\nu}_a - \frac{\bar{\nu}_z^2}{2\bar{f}_c}}{\bar{f}_c + 3\delta/2 + \frac{\bar{\nu}_z^2}{2\bar{f}_c}} + \frac{\Delta\omega}{\omega}$$

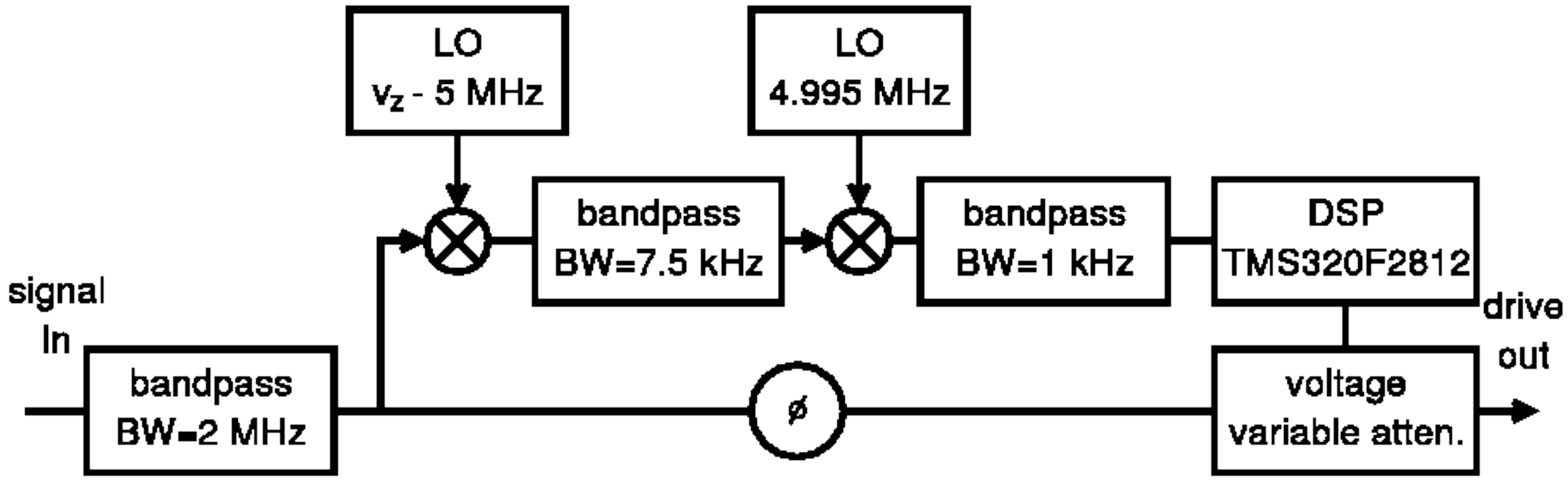
- Measure $\bar{f}_c$, $\bar{\nu}_a$, $\bar{\nu}_z$
- Calculate δ
- Calculate $\Delta\omega/\omega$

Axial Detection

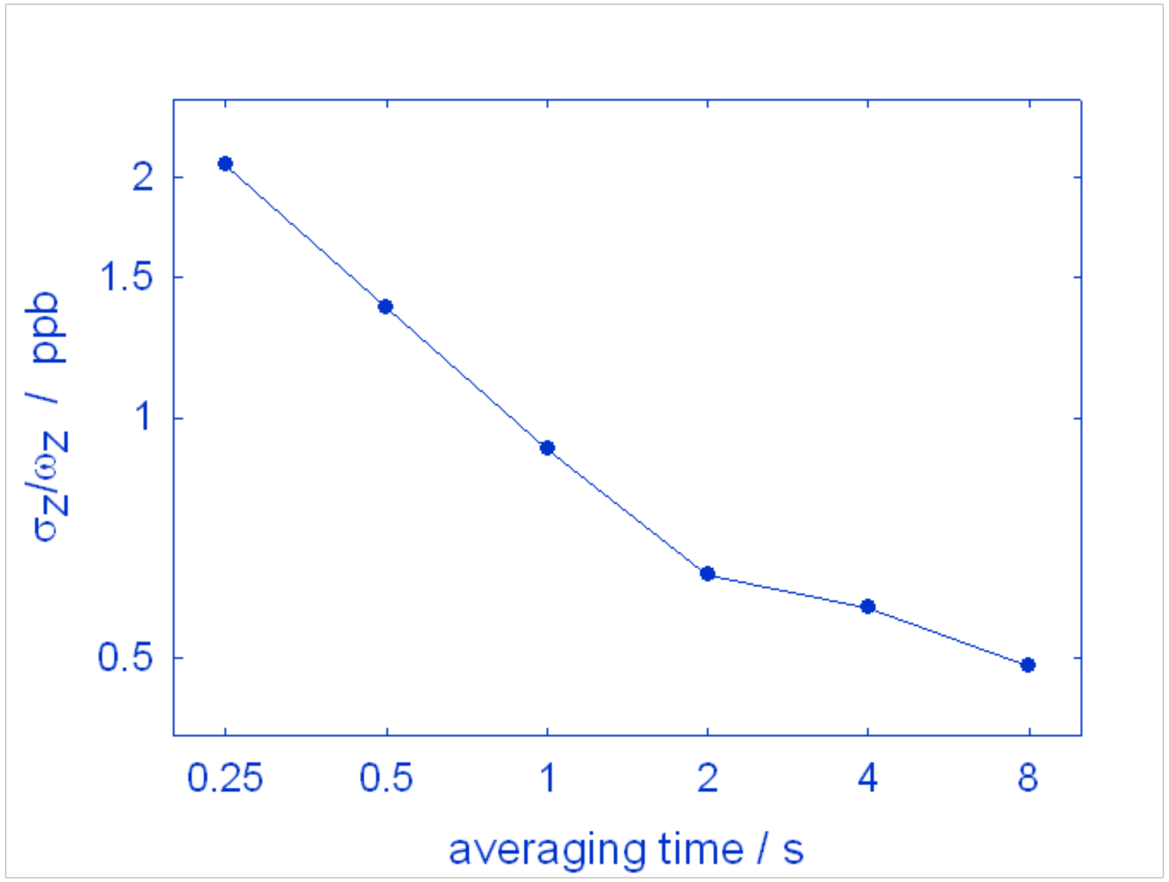




Self Excited Oscillator

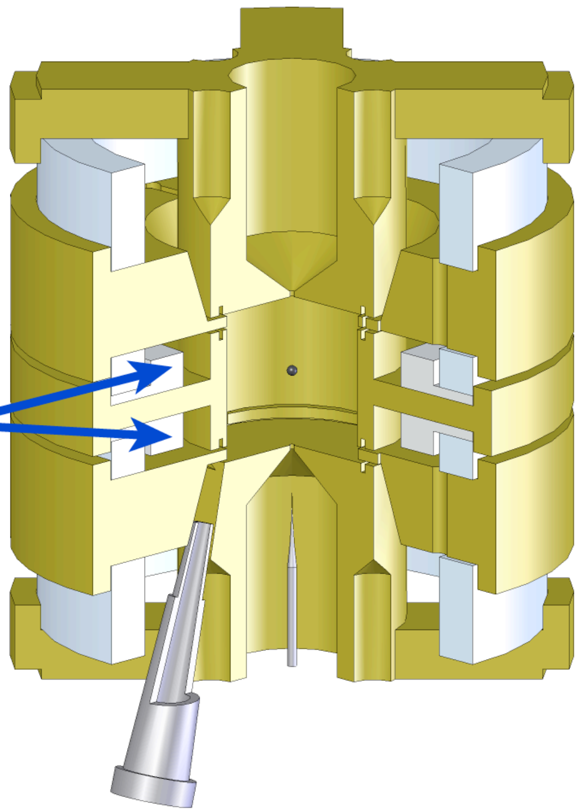


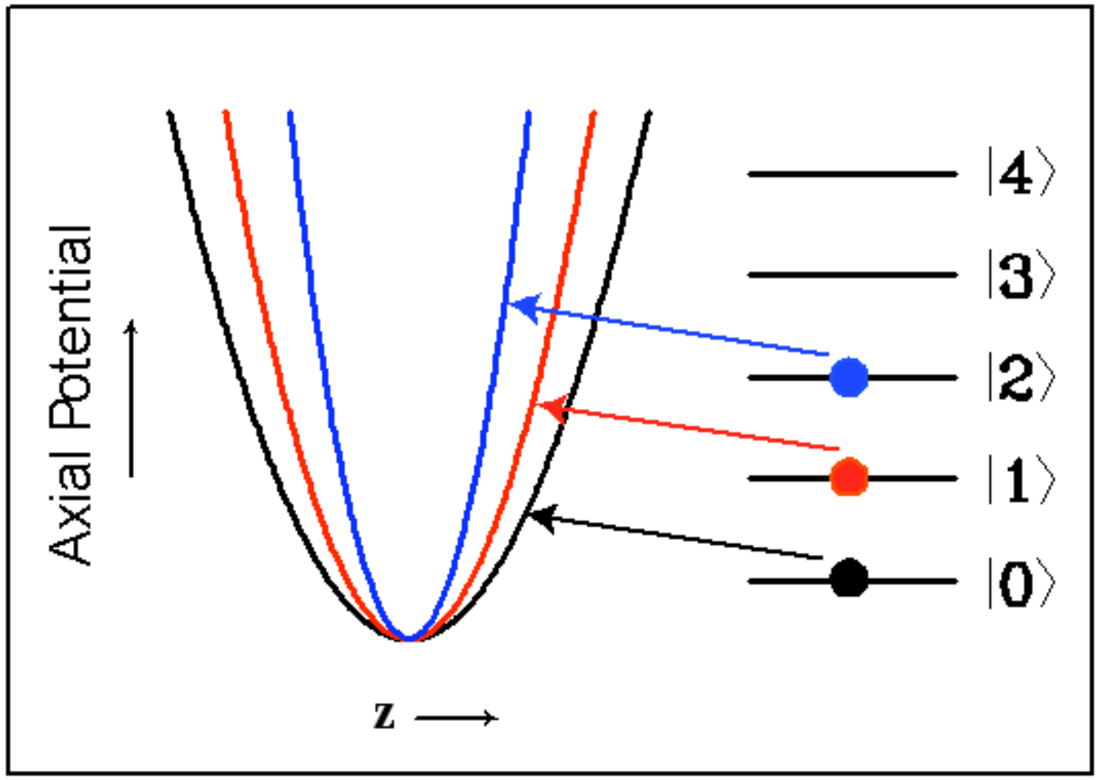
- First single-particle SEO
- Enhanced S/N



B. D'Urso, R. Van Handel, B. Odom, D. Hanneke, and G. Gabrielse, *Phys. Rev. Lett.* **94**, 113002 (2005)

Cyclotron/Spin Detection





$$H_{z0} + H'_z = \frac{1}{2} m \omega_{z0}^2 z^2 - \mu_{s,c} B_2 z^2$$

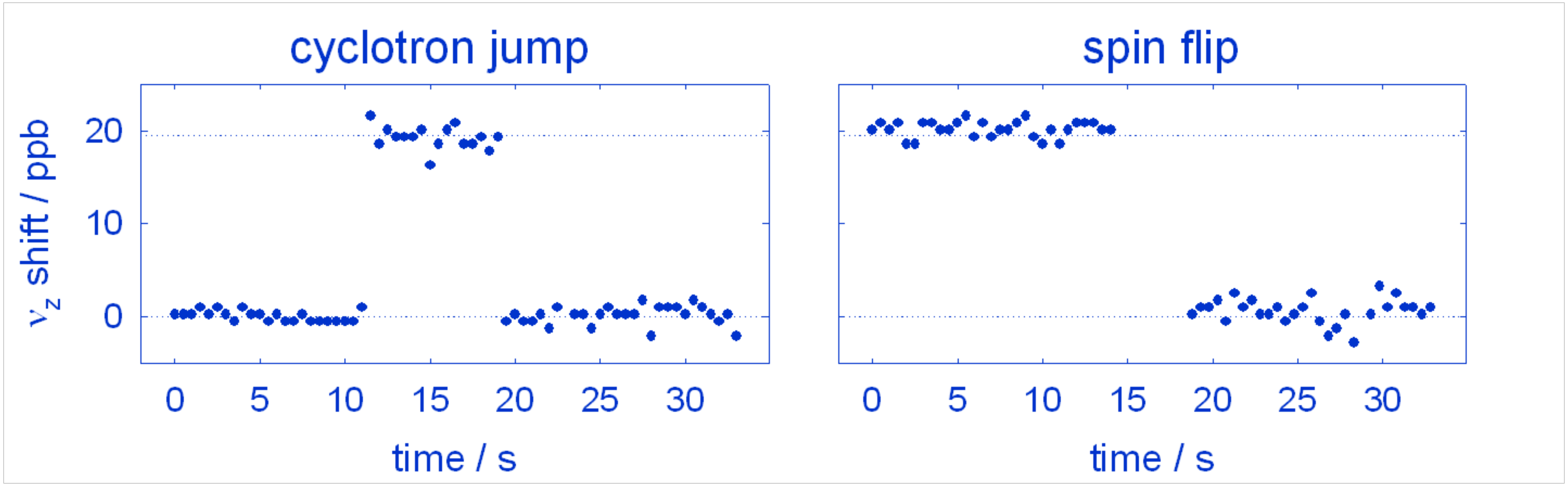
$$\Delta \vec{B} = B_2 \left[\left(z^2 - \rho^2/2 \right) \hat{z} - z \rho \hat{\rho} \right]$$

1540 T/m²

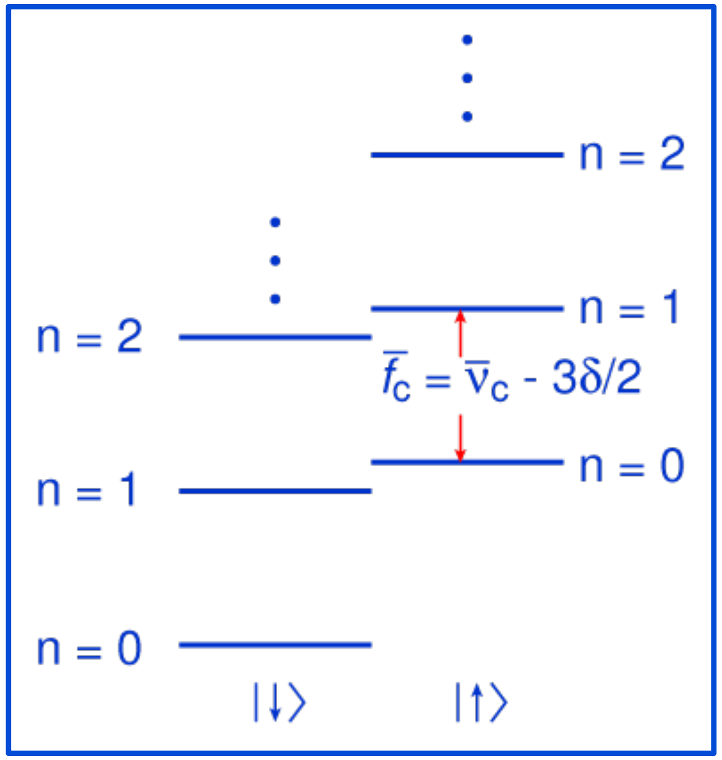
$$\frac{\Delta \nu_z}{\nu_z} \approx 2 \times 10^{-8} \left(\frac{g}{2} m_s + n \right)$$

QND measurement: $[H_c, H'_z] = 0$

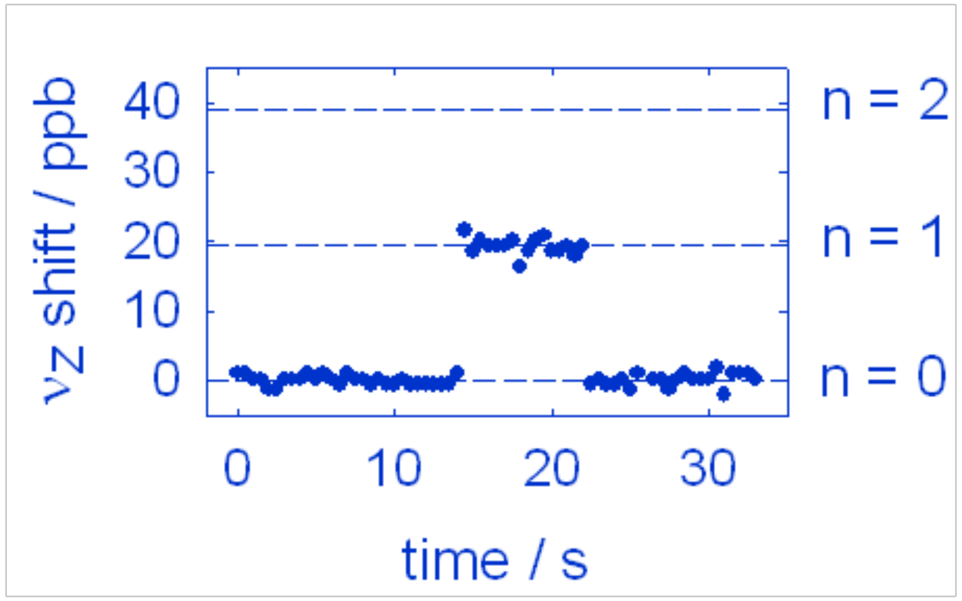
Quantum Leaps

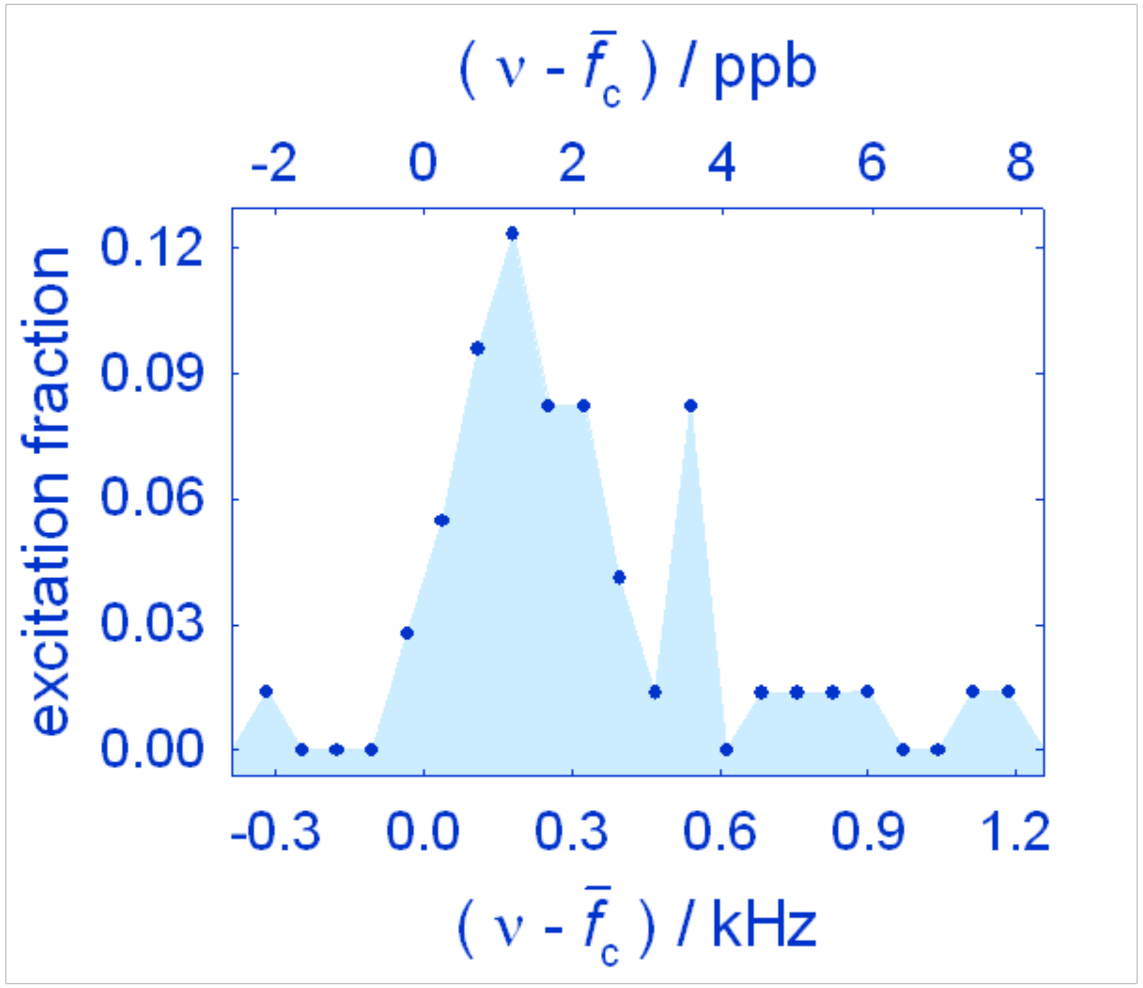


Cyclotron Procedure

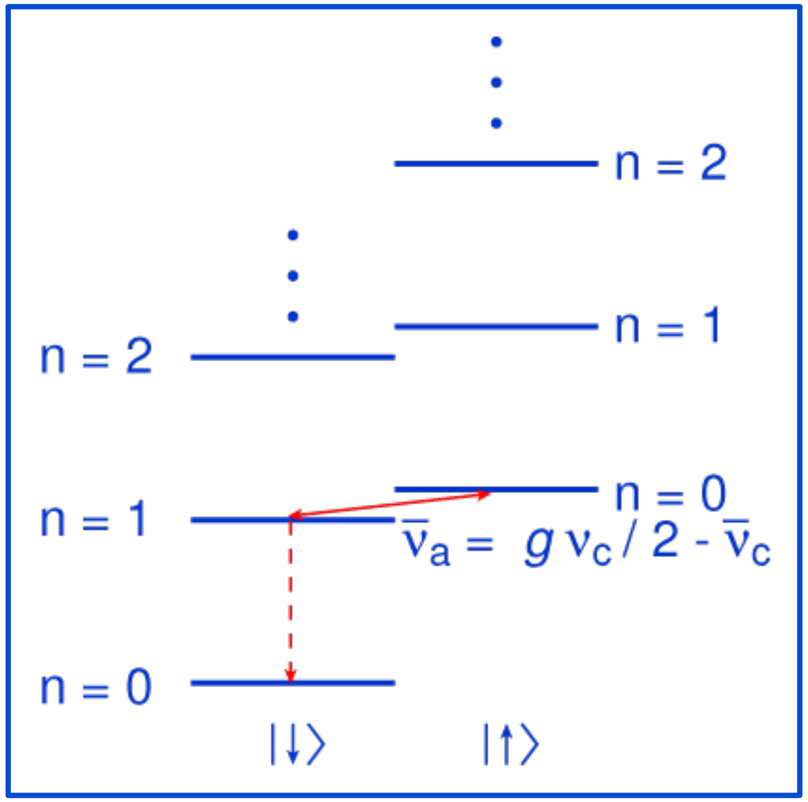


- With the e^- in the $|0,\uparrow\rangle$ state, pulse the cyclotron drive (150 GHz)
- Look for excitations to n = 1
- Make a histogram of excitations versus frequency

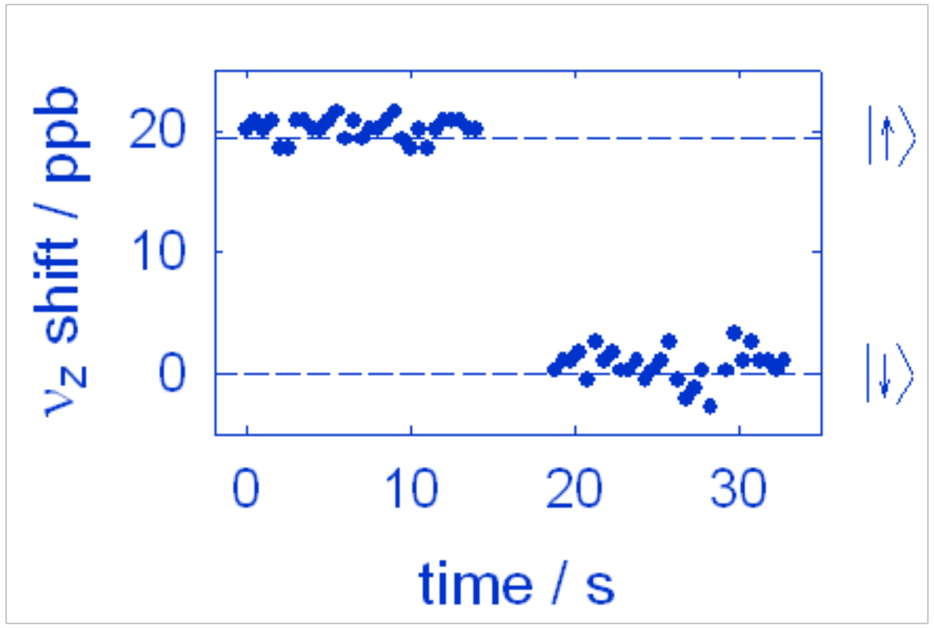


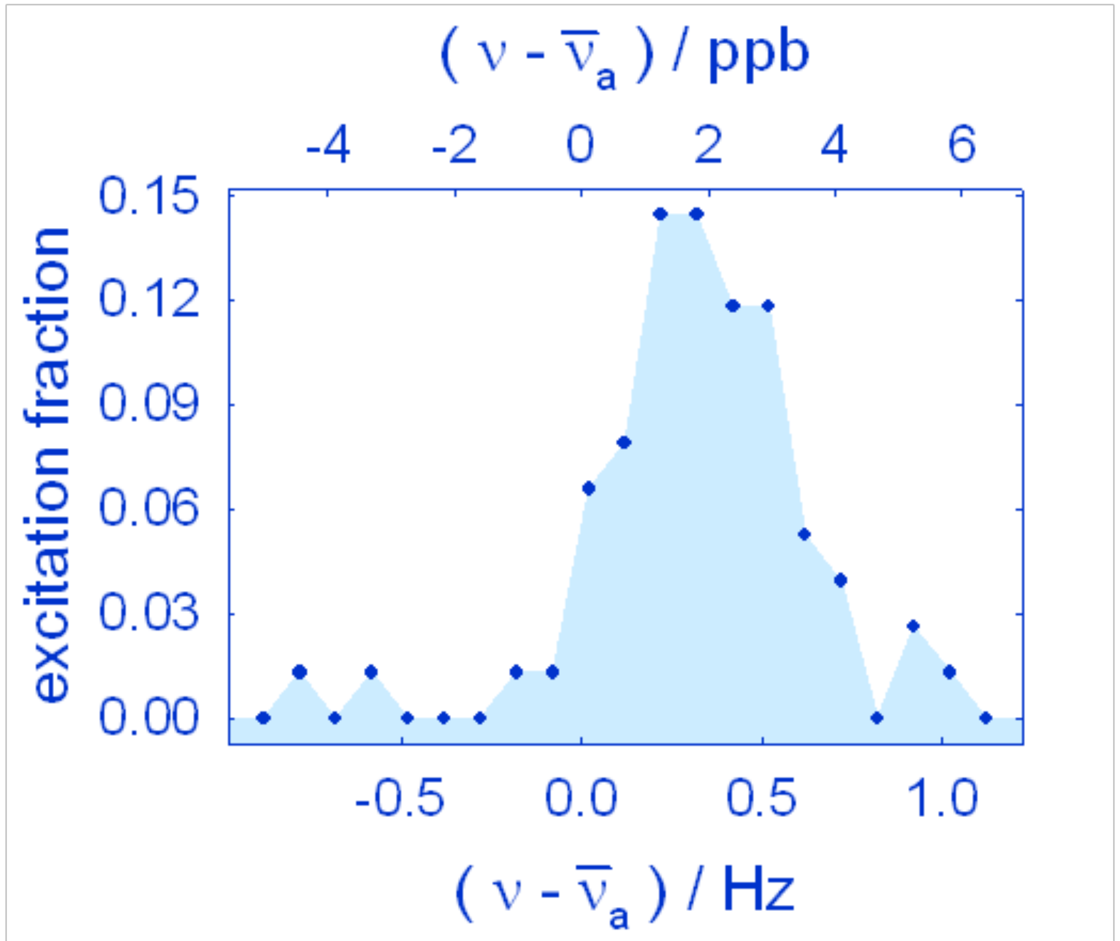


Anomaly Procedure

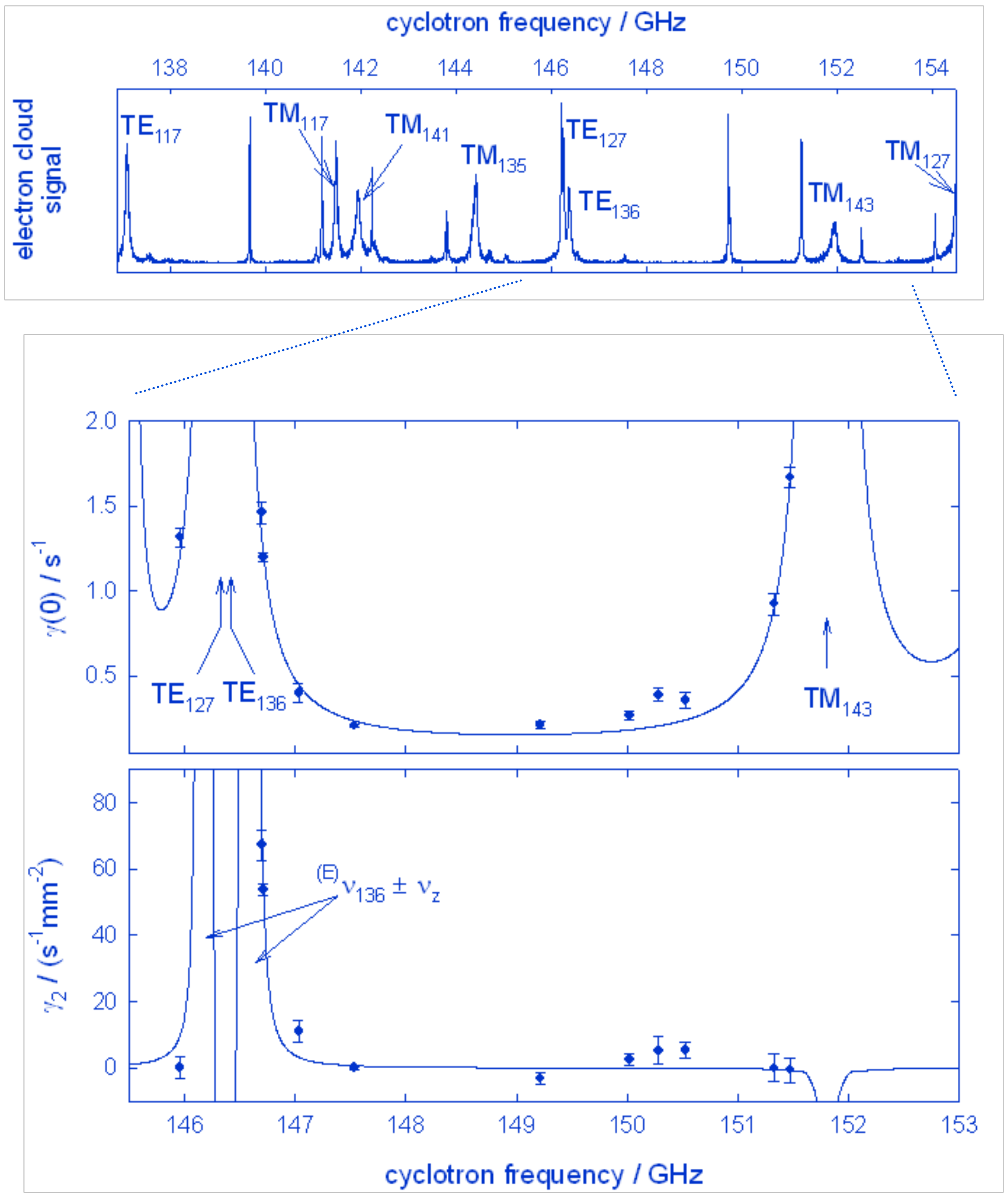


- With the e^- in the $|0,\uparrow\rangle$ state, pulse the anomaly drive (174 MHz) to move the e^- through the gradient $\Delta\vec{B} \sim z\rho\hat{\rho}$
- Look for a decay to $|0,\downarrow\rangle$ (slow!)
- Make a histogram of spin flips versus frequency
- Pump the electron back to $|0,\uparrow\rangle$ state before each measurement.



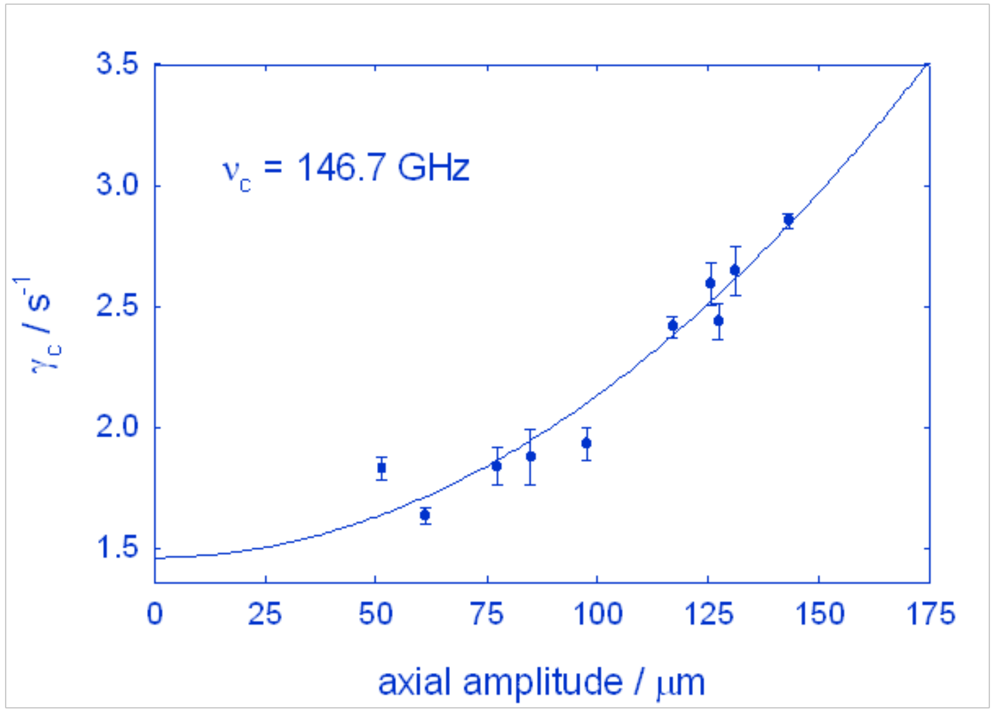


Determining the Cavity Shifts



Two mode-detection techniques

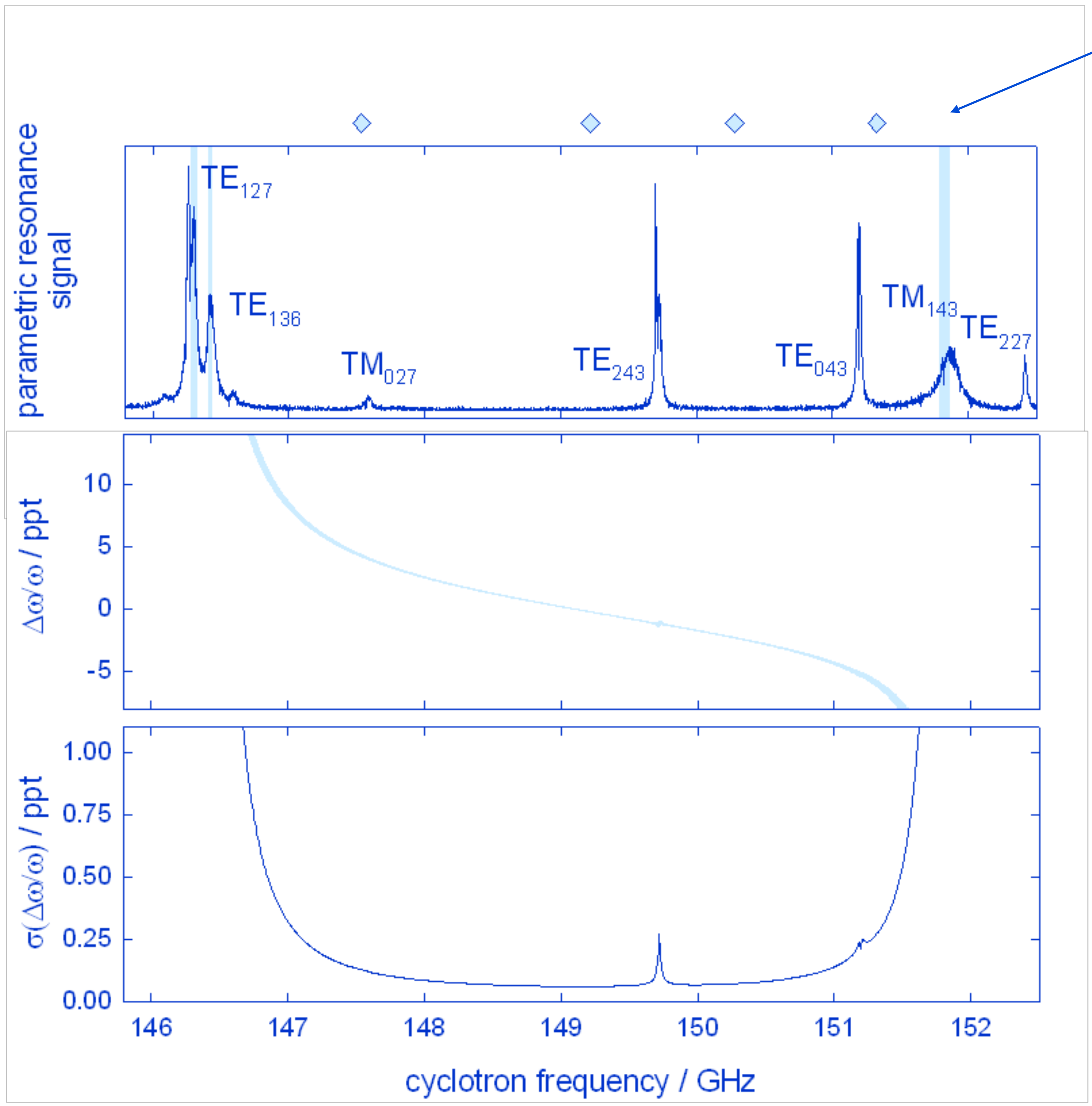
- A synchronized cloud of electrons
- The single electron itself



Synchronized electron technique:
J. Tan and G Gabrielse, *Phys. Rev. Lett.* **67**, 3090 (1991)

Determining the Cavity Shifts

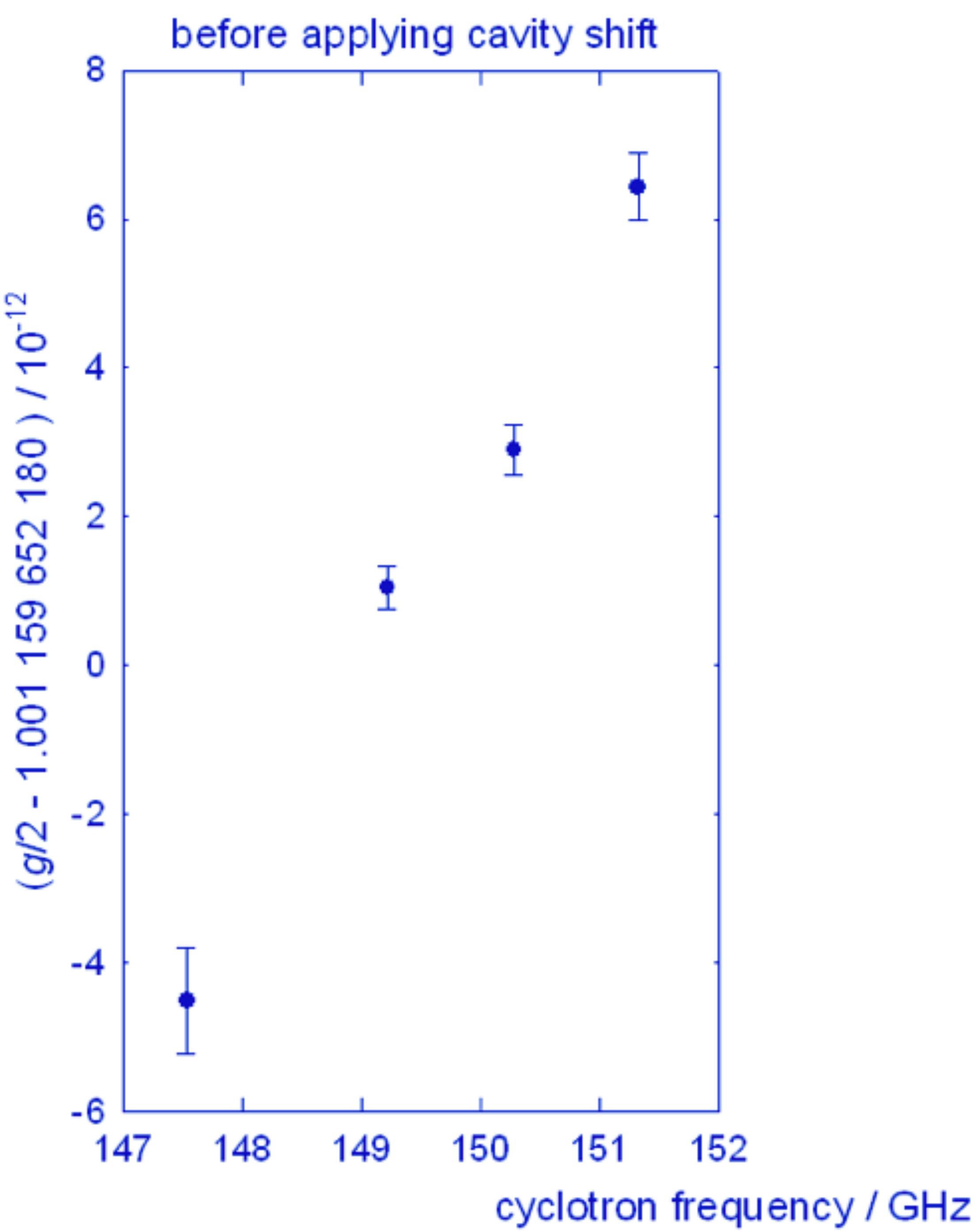
locations of our g-value measurements



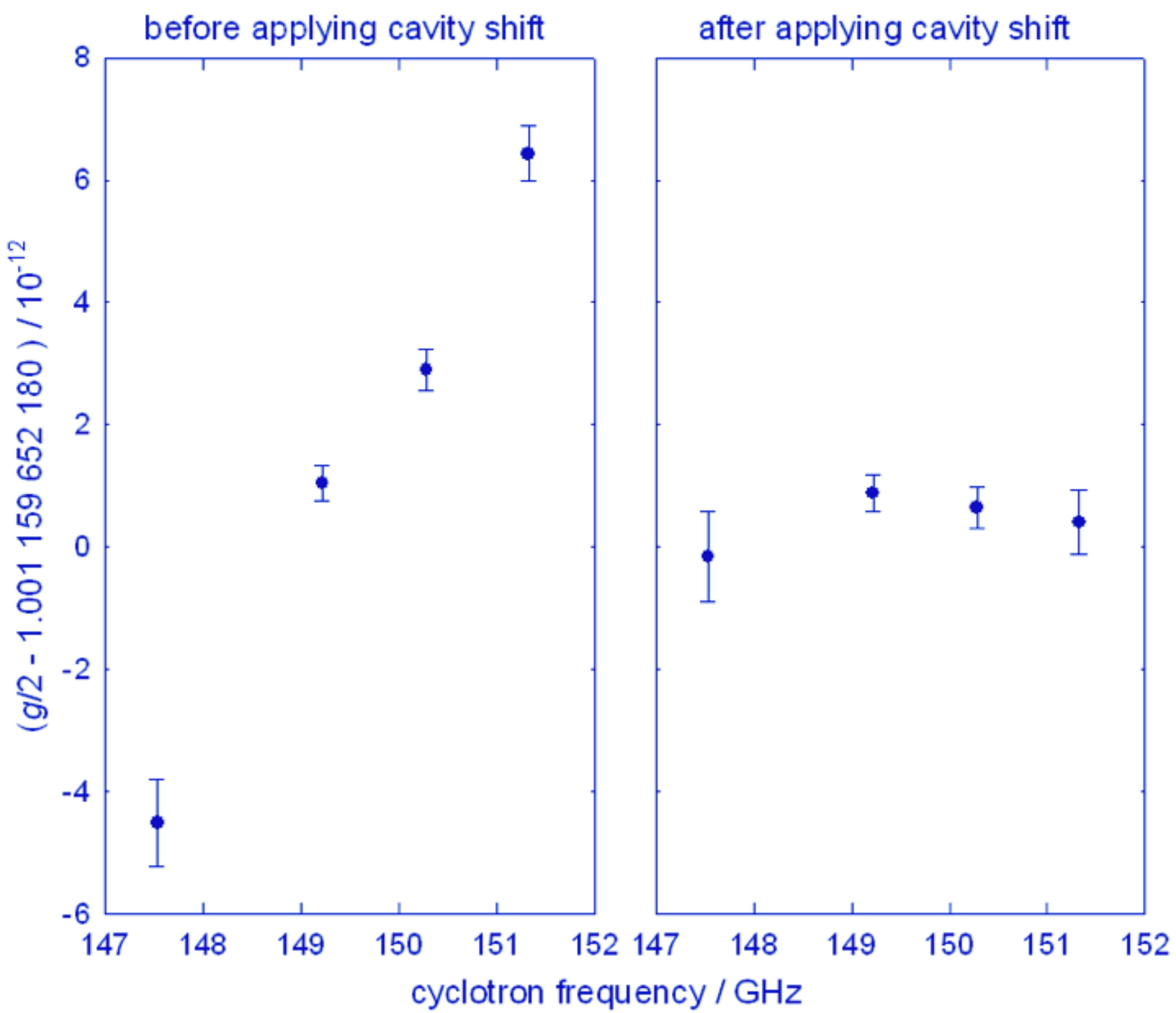
- Determine the locations of the coupled modes
- Calculate the *g* value shift
- Uncertainty in mode location → uncertainty in *g*

L.S. Brown *et al.*, *Phys. Rev. A* **32**, 3204 (1985)

Applying the Cavity Shifts



Applying the Cavity Shifts



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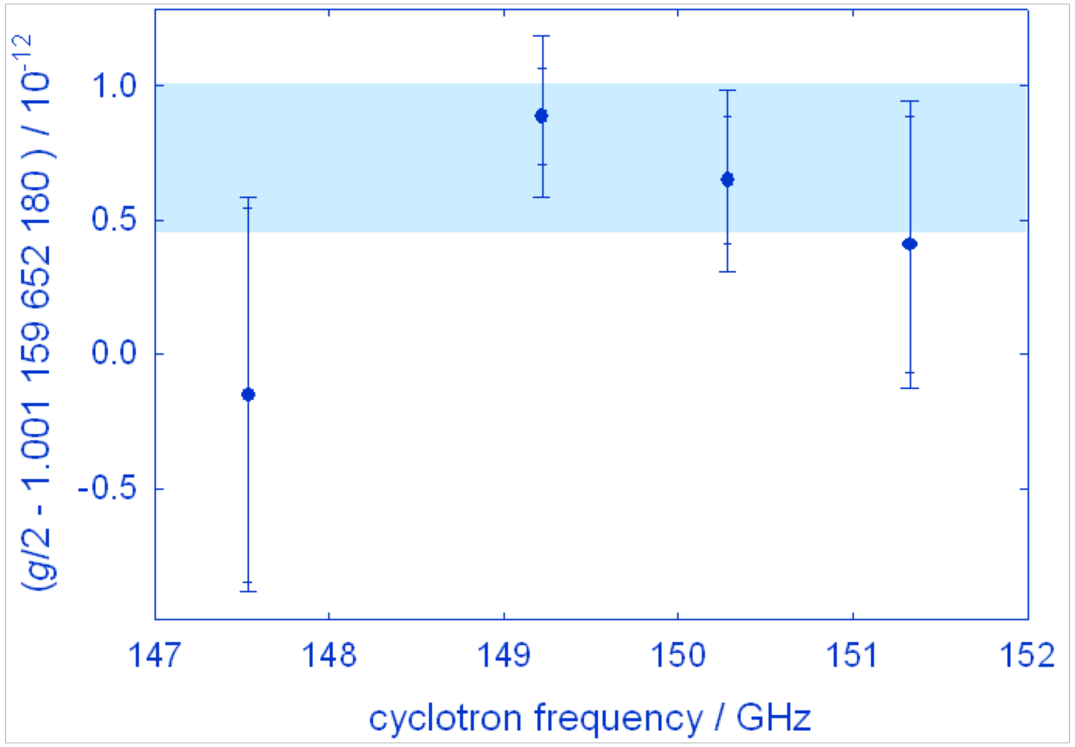
Uncertainties

g/2 = 1.001 159 652 180 73 (28) [0.28 ppt]



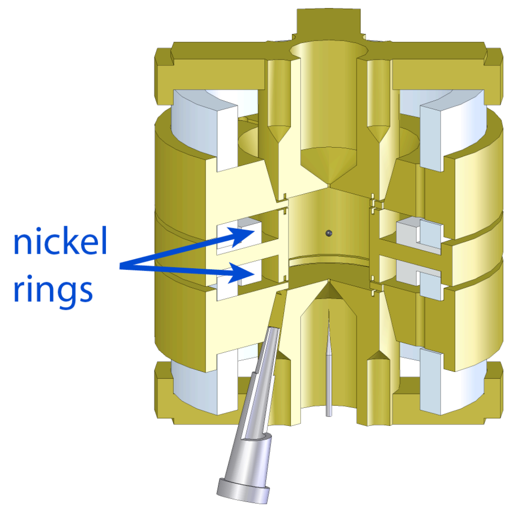
Uncertainties for *g* in parts-per-trillion.

ν_c / GHz =	147.5	149.2	150.3	151.3
Statistics	0.39	0.17	0.17	0.24
Cavity shift	0.13	0.06	0.07	0.28
Uncorrelated lineshape model	0.56	0.00	0.15	0.30
Correlated lineshape model	0.24	0.24	0.24	0.24
Total	**0.73**	**0.30**	**0.34**	**0.53**

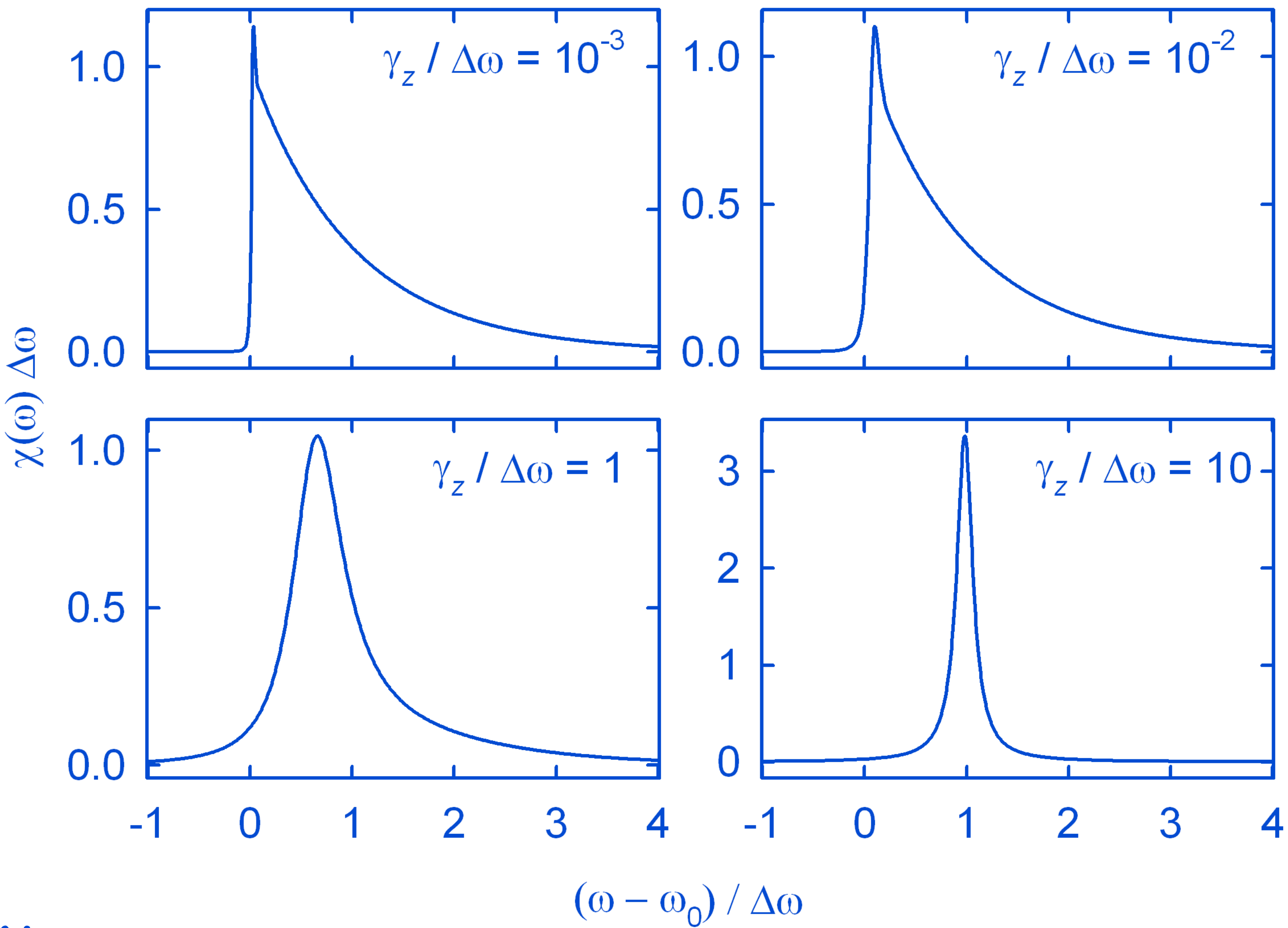
Ideal Lineshapes

cyclotron or anomaly

$$\omega(z) = \omega_0 \left(1 + \frac{B_2}{B} z^2\right)$$



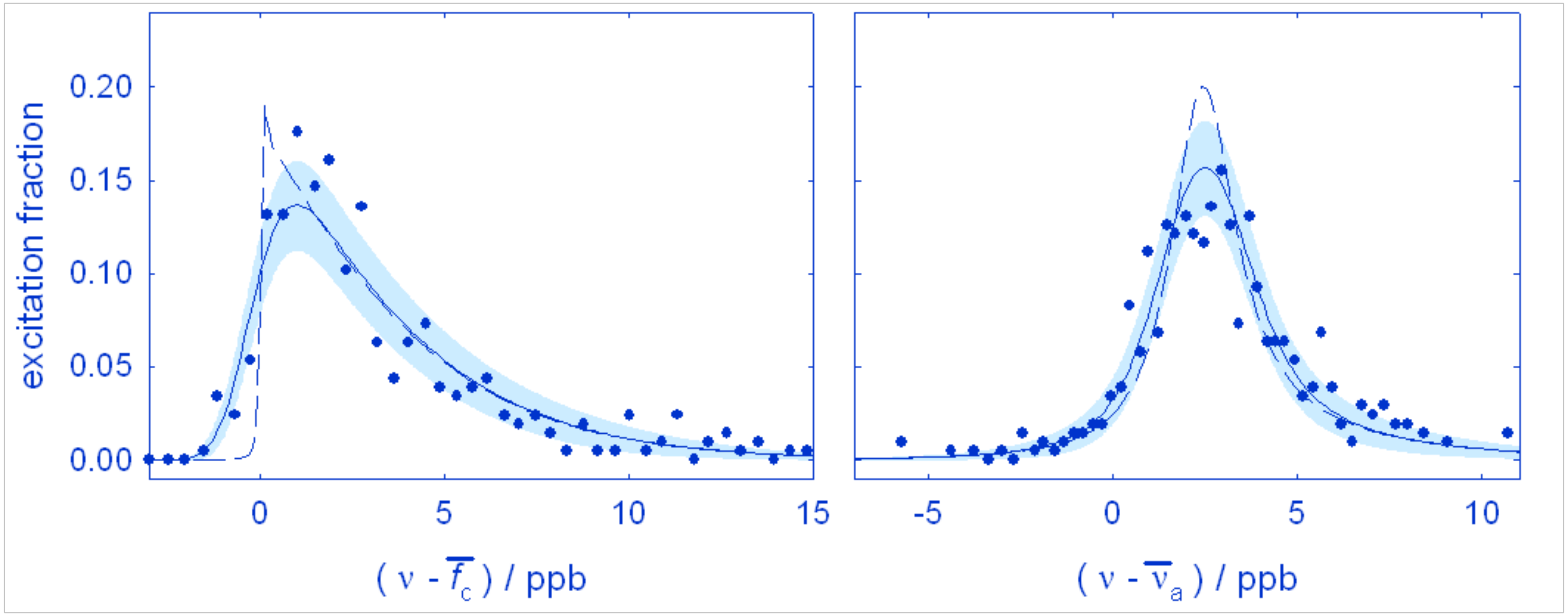
$$\Delta\vec{B} = B_2 \left[\left(z^2 - \rho^2/2\right) \hat{z} - z\rho\hat{\rho} \right]$$



L.S. Brown, *Ann. Phys. (N.Y.)* **159**, 62 (1985)

- Brownian motion of *z*
- Shape depends on axial damping (γ_z) and frequency (ω_0)
- Mean is independent of γ_z

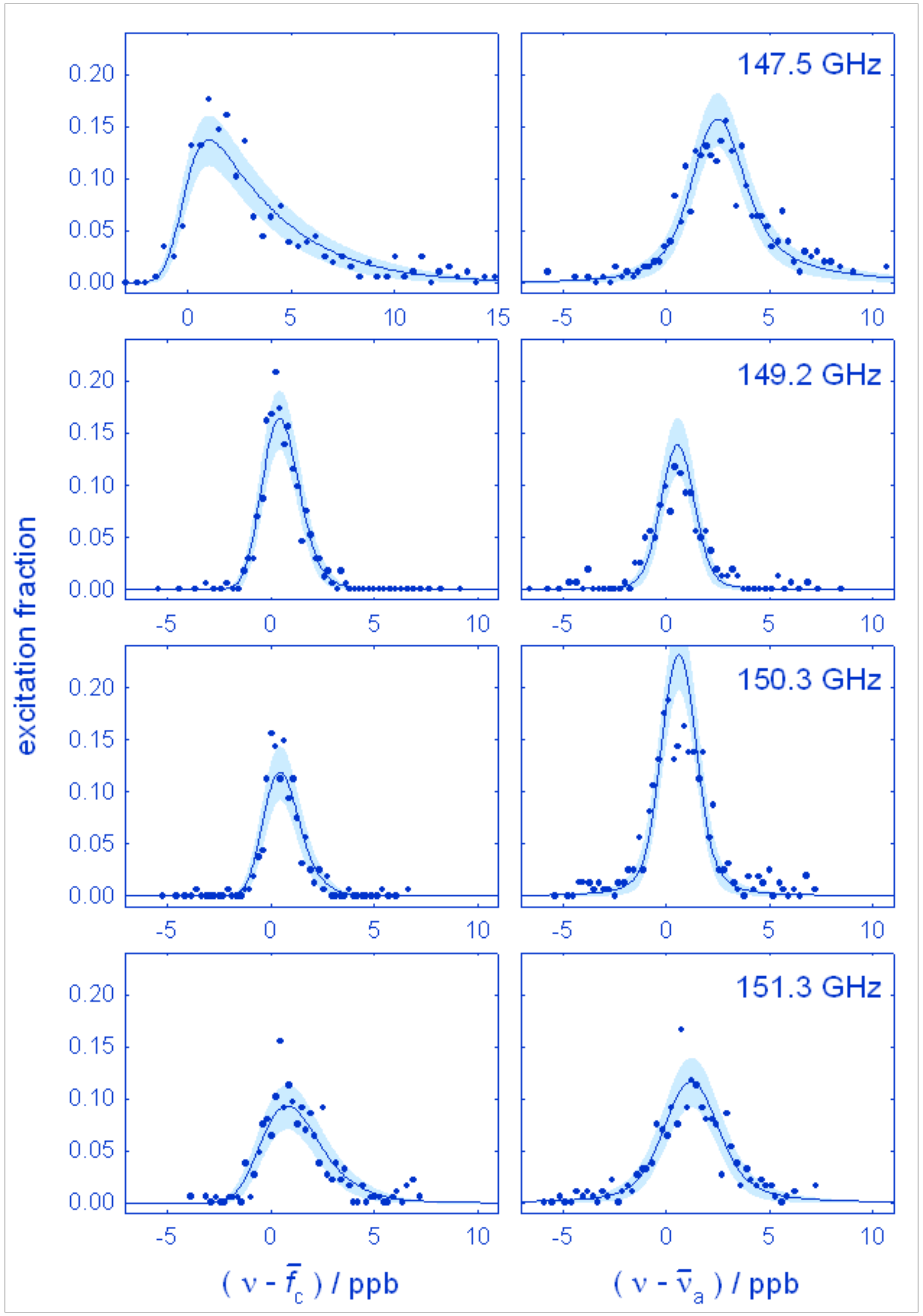
Lineshape Model Uncertainty

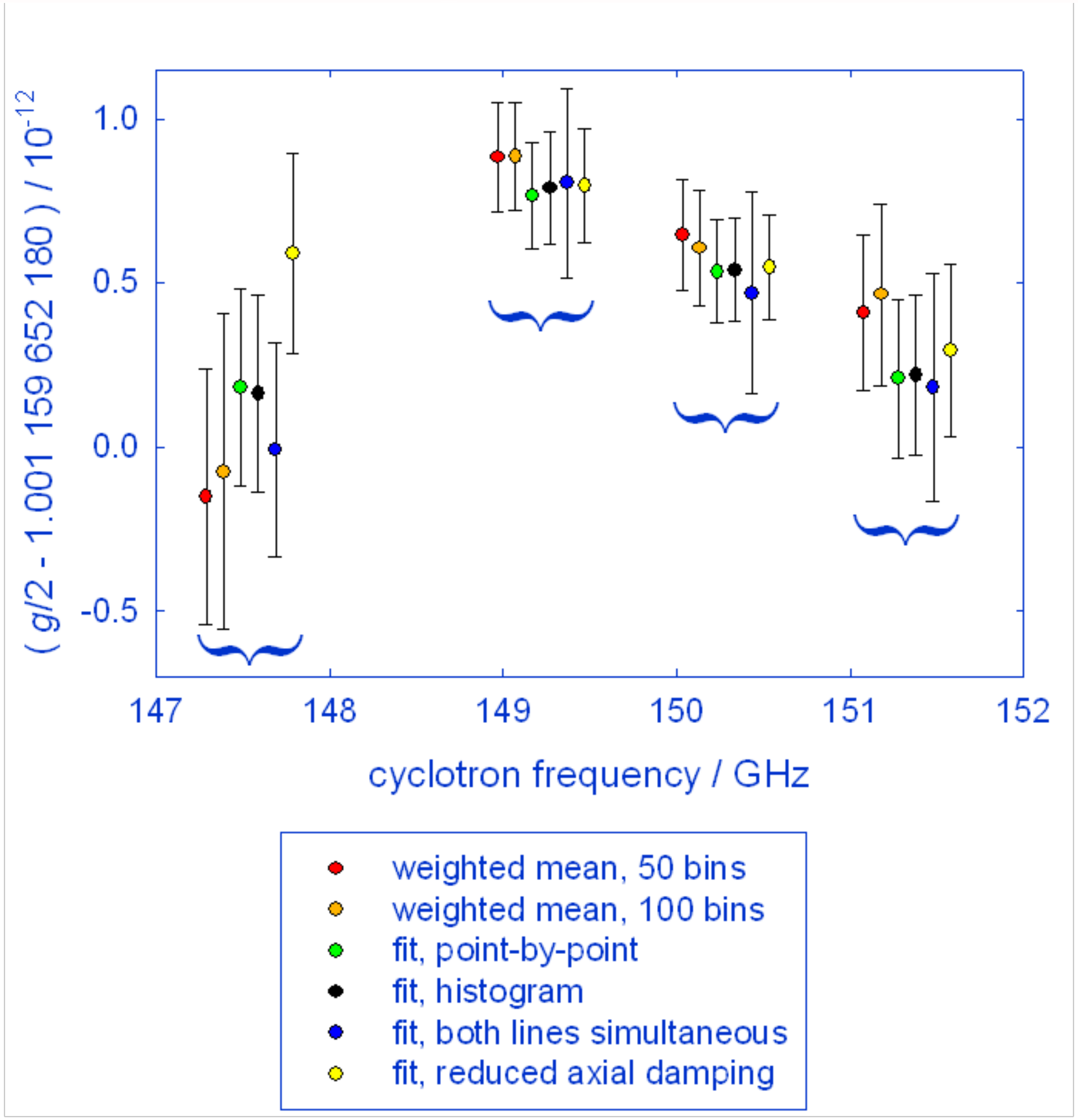


- Concern about magnetic field noise
- Get $\bar{f}_c$, $\bar{\nu}_a$ with a weighted-mean method
- Check with fits (assuming a noise model)
- How well do they agree?

self-calibrating!

Lineshape Model Uncertainty





Outline

I. Introduction
II. Measurement overview
III. Novel techniques
IV. Measurement details
V. Uncertainties
VI. What's next

Lepton Moments

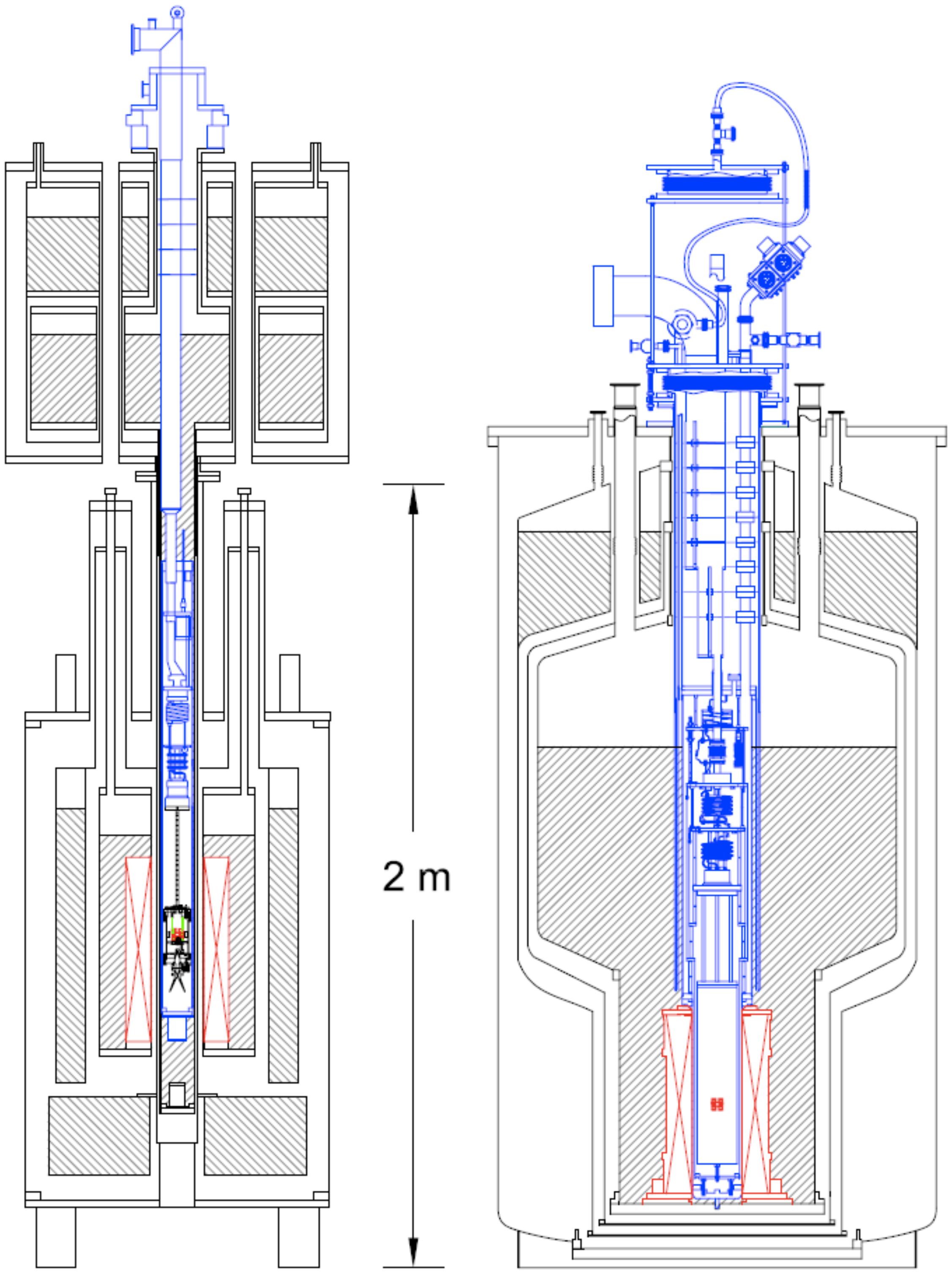
- e^+ *g*-value / CPT test
- Cavity-assisted axial-cyclotron sideband cooling and other line-narrowing techniques
- Speed up the measurement cycle (π-pulses and adiabatic fast passage)

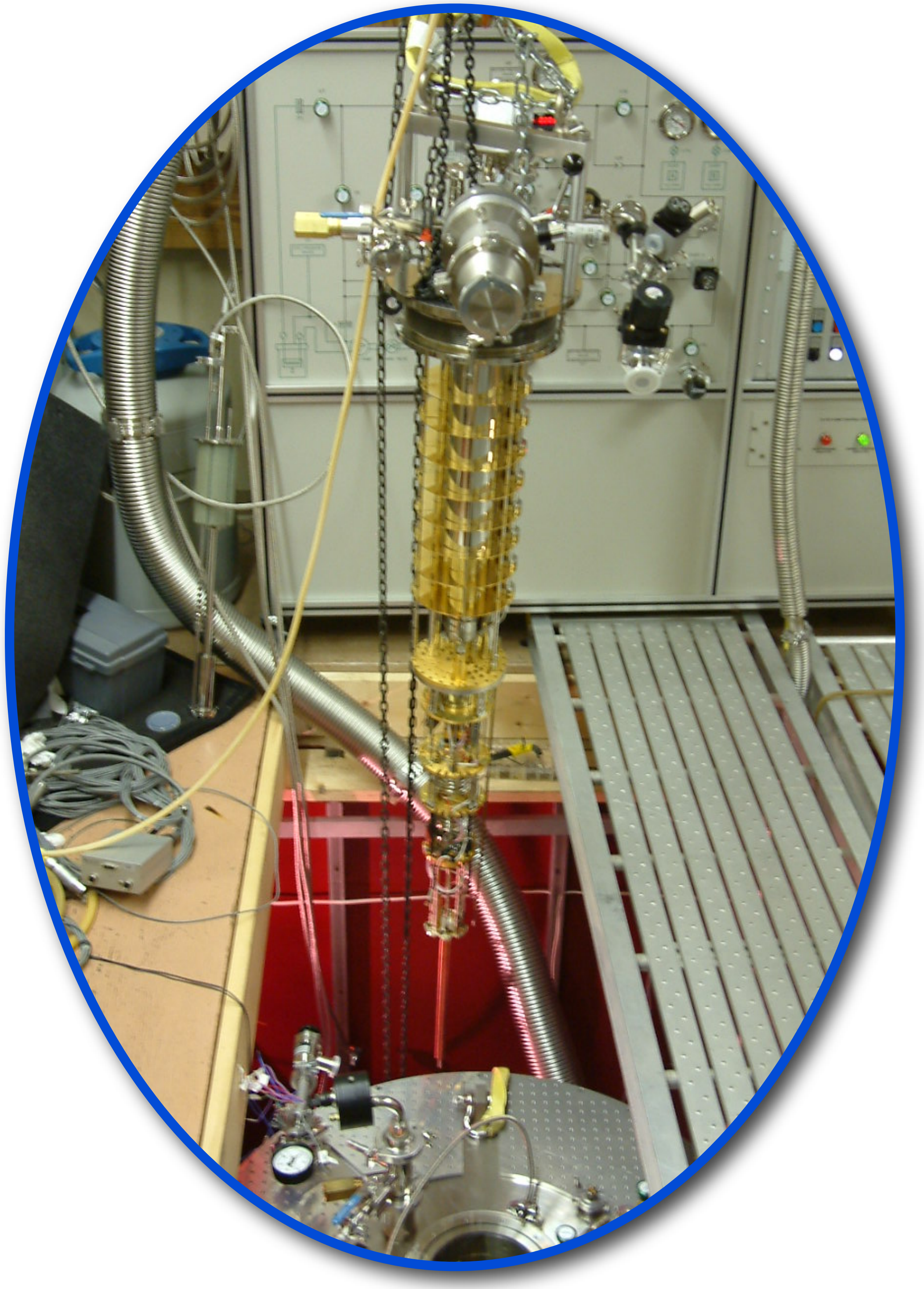
Shannon Fogwell Hoogerheide

Josh Dorr

Poster session
Wednesday 4pm

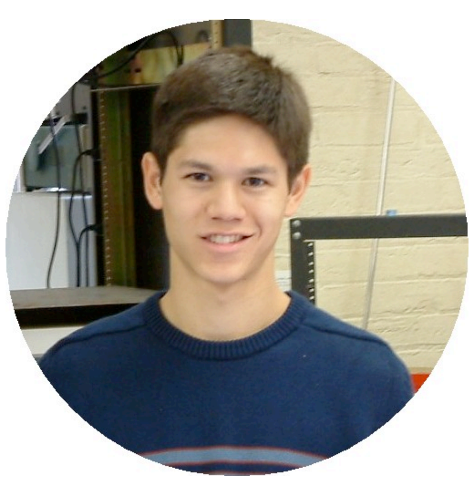
New High-Stability Apparatus





Baryon Magnetic Moments

- μ_p, $\mu_{\bar{p}}$ (first single-proton spin flip, improve the antiproton measurement by 10^6)
- Harvard: N. Guise, J. DiSciacca, and G. Gabrielse, *Phys. Rev. Lett.* **104** 143001 (2010)
- GSI–Mainz: C. C. Rodegheri, *et al*, *Hyperfine Int.* **194** 93–98 (2009)



Nick Guise

Jack DiSciacca

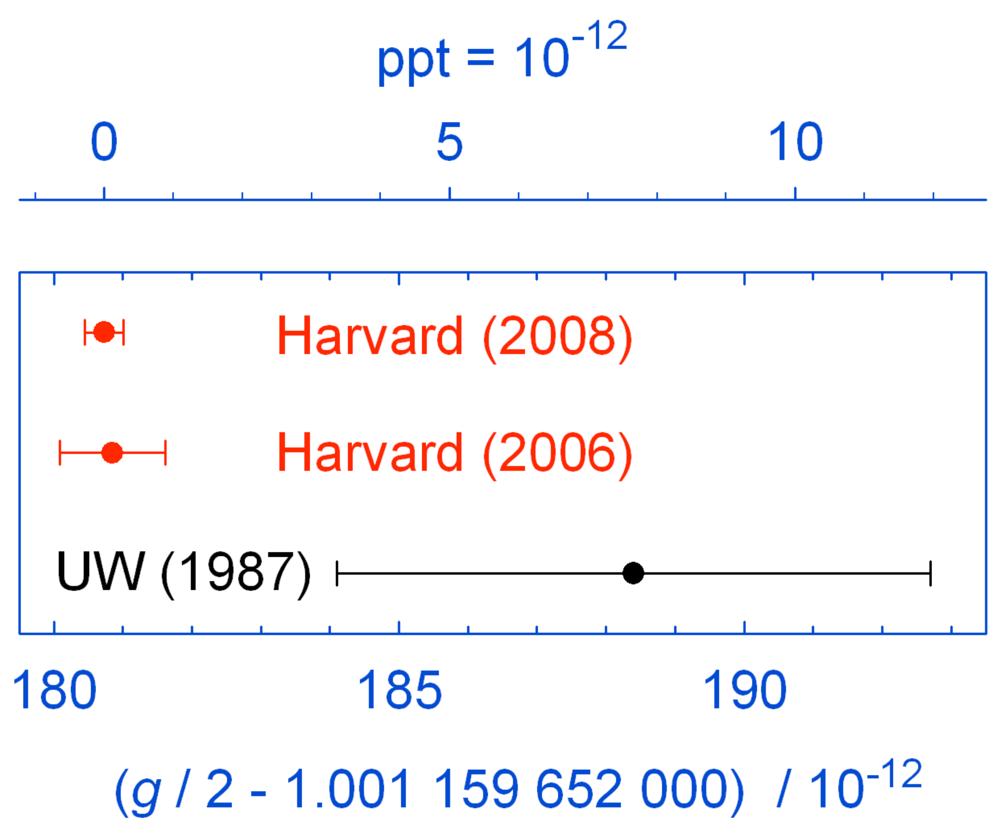
Next talk

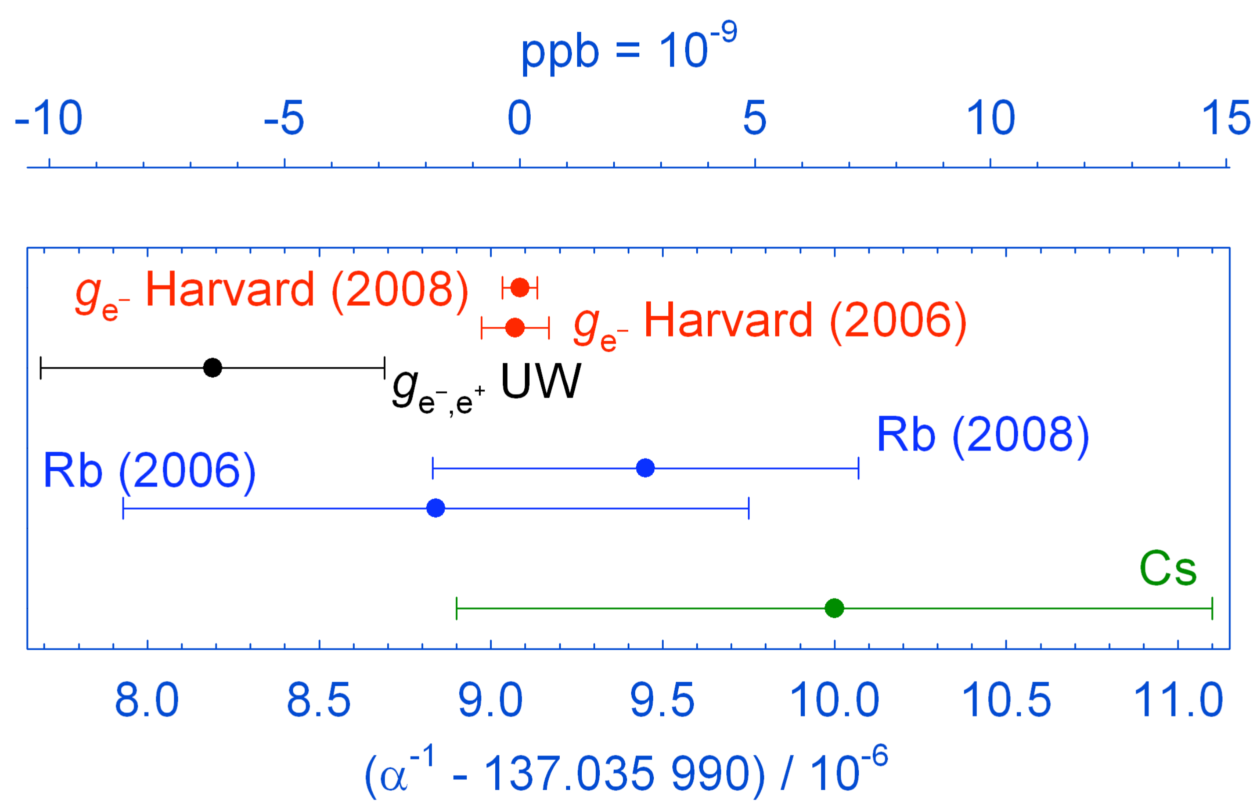
Summary

- First improvements in *g* in 20 years (factor of 15 total)
- Cavity effects no longer dominate the uncertainties
- More to come...

α^{-1} = 137.035 999 084 (51) [0.37 ppb]
= 137.035 999 084 (33) (39)
[0.24 ppb, exp.][0.28 ppb, th.]

$g/2$ = 1.001 159 652 180 73 (28) [0.28 ppt]





D. Hanneke, S. Fogwell, and G. Gabrielse, *Phys. Rev. Lett.* **100**, 120801 (2008)

B. Odom, D. Hanneke, B. D'Urso, and G. Gabrielse, *Phys. Rev. Lett.* **97**, 030801 (2006)

G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, and B. Odom, *Phys. Rev. Lett.* **97**, 030802 (2006). *Ibid*. **99** 039902(E) (2007)