Toward an Improved Electron *g*-2 Measurement

David A. Hanneke, Brian C. Odom and Gerald Gabrielse
Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

Measurements of the electron *g*-factor, combined with an independent measurement of the fine structure constant, represent the most precise test of QED. Alternately, assuming the validity of QED, the *g*-factor measurement provides the most precise determination of the fine structure constant.

The highest precision *g*-factor measurement published to date, at 3.8 parts per billion, was performed using classical cyclotron spectroscopy in a hyperbolic Penning trap at a temperature of 4 K. We use single quantum cyclotron spectroscopy in a cylindrical Penning trap at 100 mK. With these techniques, we hope to achieve at least an order of magnitude higher precision. We can repeatedly measure the *g*-factor to a precision of 1 ppb and are analyzing systematic effects. The leading-order systematic of the previous measurement was cavity shifts; our cylindrical trap allows for easy calculation of the cavity modes. These calculations, when combined with actual measurements of the modes, allow us to greatly reduce that effect.

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The anomalous magnetic moment

Classical, non-relativistic → $g = 1$

Dirac equation as single-particle wave equation → $g = 2$

$$g \equiv \frac{\mu / \mu_{\mathrm{B}}}{\mathrm{S} / \hbar}$$

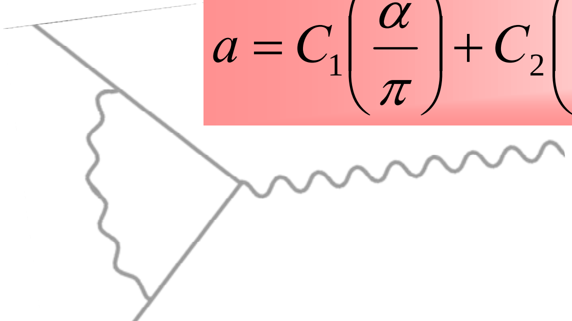
Quantum Electrodynamics (QED) → $g = 2.002\,319\,304\ldots$

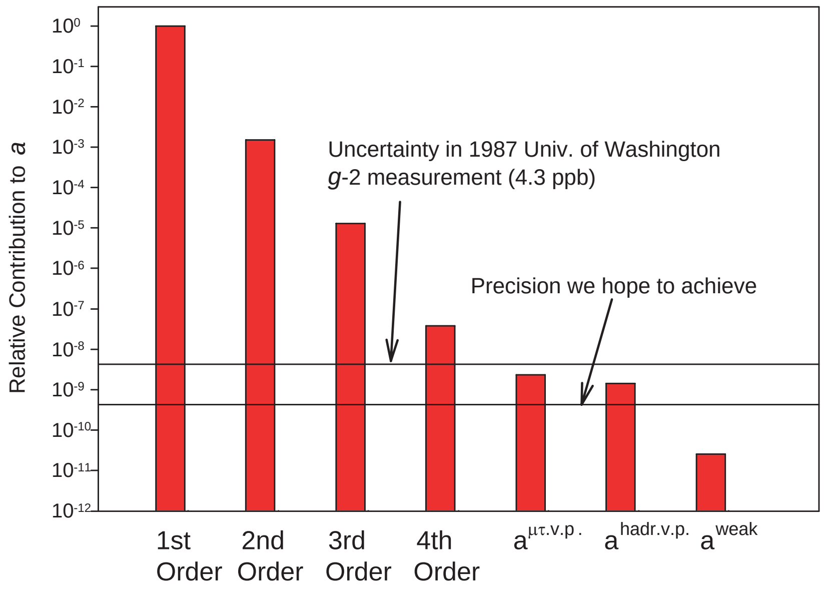
$$a \equiv \frac{g-2}{2} \approx 10^{-3}$$

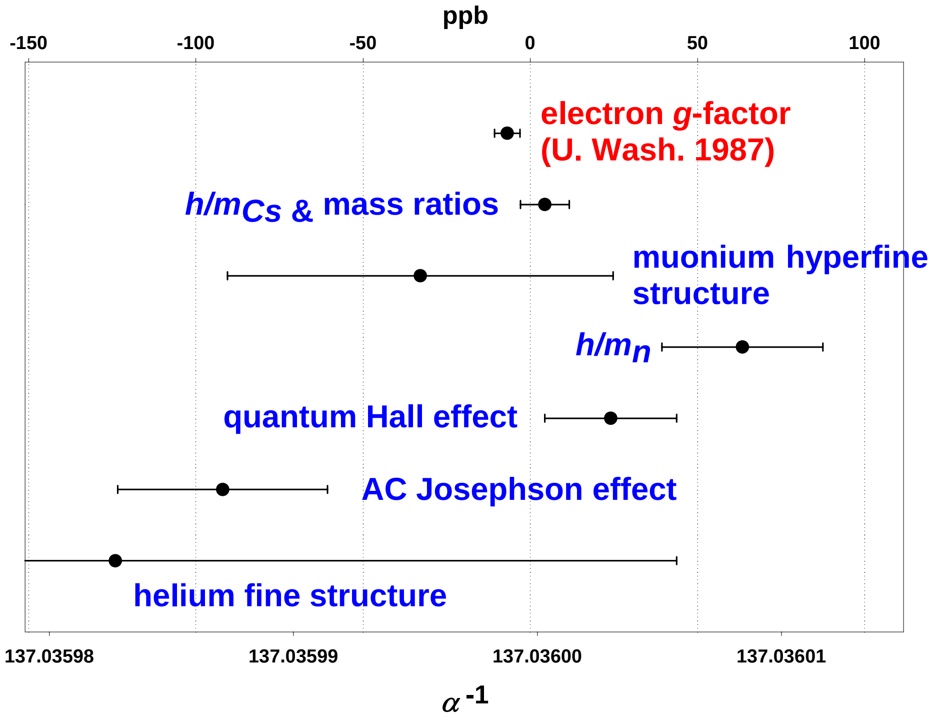
QED and *g*-2

The standard model predicts that the anomaly, *a*, can be expanded in powers of the fine structure constant, α.

$$a = C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + \ldots + a^{\mu\tau.vac.pol.} + a^{hadronic.v.p.} + a^{weak}$$

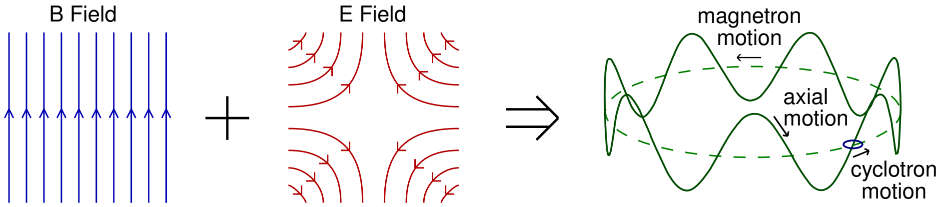




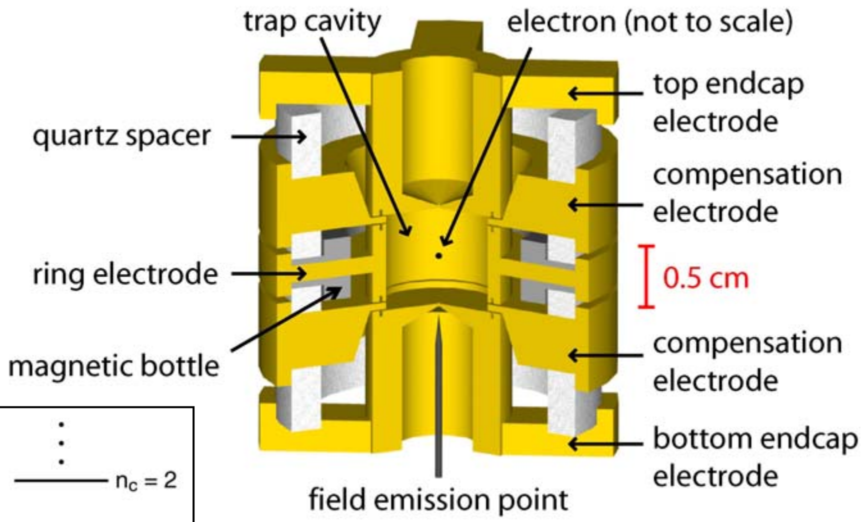


The agreement, or lack thereof, among measurements of α serves as a test of QED.

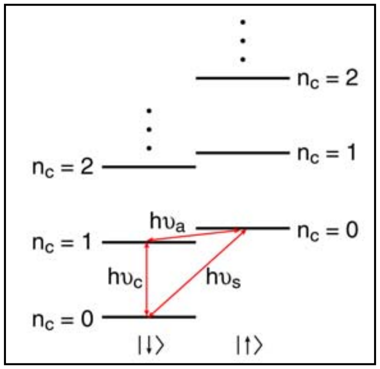
The Penning Trap

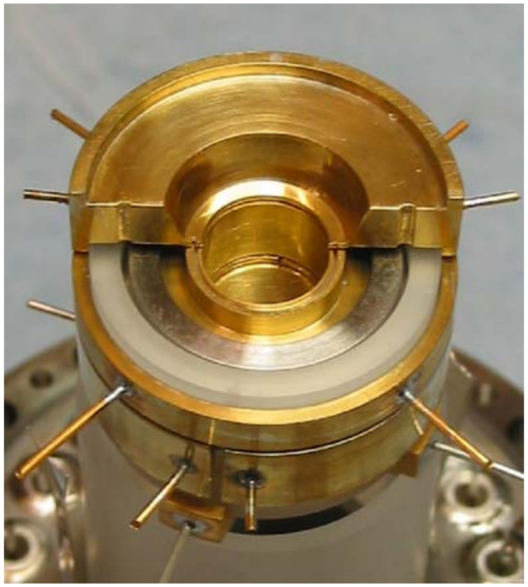


A uniform magnetic field confines the electron radially, while a static quadrupole electric field confines it axially. The slow magnetron motion is caused by the electron's **E** x **B** drift.



motion	frequency	$h\upsilon/k_b$	damping
axial	200 MHz	10.0 mK	1 Hz
cyclotron	149.0 GHz	7.2 K	0.02 Hz
spin	149.2 GHz	7.2 K	10^{-12} Hz
magnetron	134 kHz	0.64 μK	10^{-15} Hz





g-2 in a Penning Trap

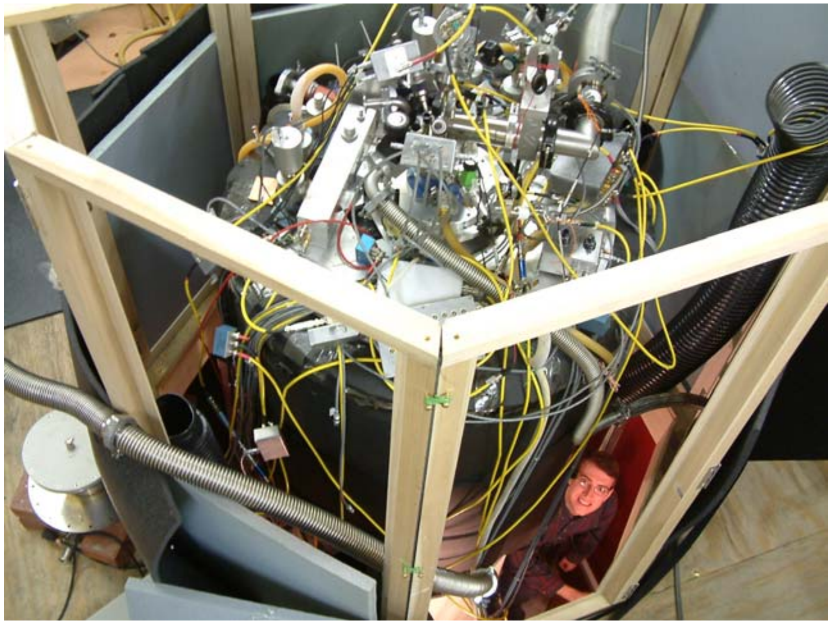
In free space, the anomaly is just the ratio of the anomaly frequency to the cyclotron frequency. ($\nu_a = \nu_s - \nu_c$)

$$a = \frac{\nu_a}{\nu_c}$$

In a Penning trap, however, the electric field perturbs the magnetic motion, resulting in a correction to the measured anomaly and cyclotron frequencies.

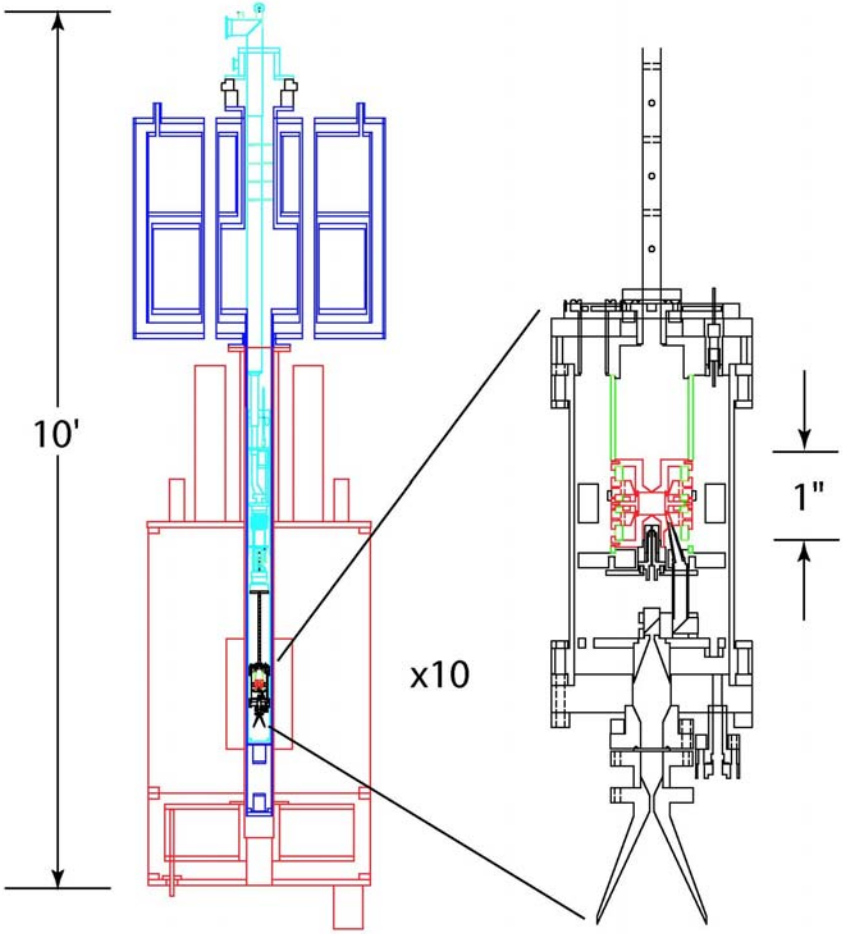
$$a = \frac{\bar{\nu}_a - \bar{\nu}_z^{\,2}/2\bar{\nu}_c}{\bar{\nu}_c + \bar{\nu}_z^{\,2}/2\bar{\nu}_c}$$

Apparatus



A 5.3 T superconducting magnet provides the field for the Penning trap. A dilution refrigerator keeps the trap electrodes at 100 mK.

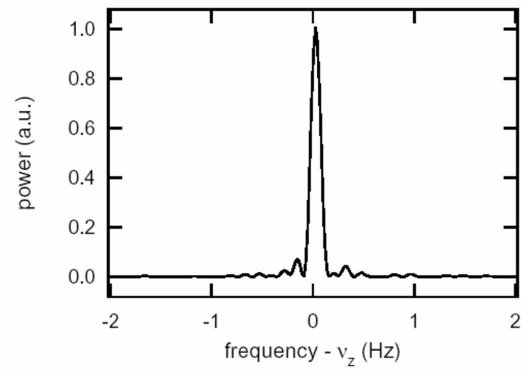
It is a tabletop experiment…provided you have a high enough ceiling.

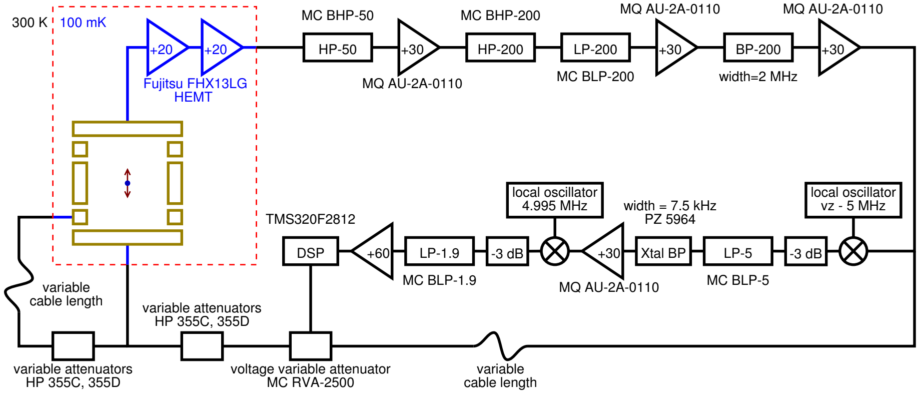


Detection

…of the Axial Motion

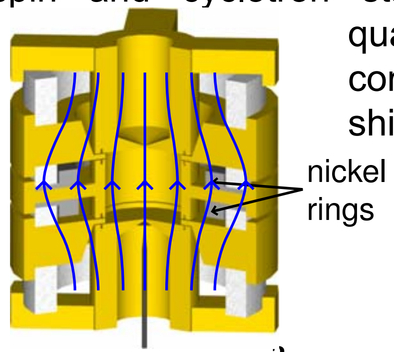
The electron’s axial motion induces image currents on the trap electrodes. By coupling these currents to a tuned circuit and amplifying them, we can measure the frequency and relative amplitude of this motion. By positively feeding the detected signal back to the electron, We can measure the axial frequency to better than 1 Hz in 200 MHz.





…of the Cyclotron State

The 149 GHz cyclotron frequency is too high for direct detection. We use nickel rings to introduce a quadratic perturbation in the magnetic field. This perturbation makes the axial potential well (and thus the axial frequency) depend on the total magnetic moment of the electron and thus on the spin and cyclotron states. For our bottle, a quantum jump corresponds to a 4 Hz shift in axial frequency.

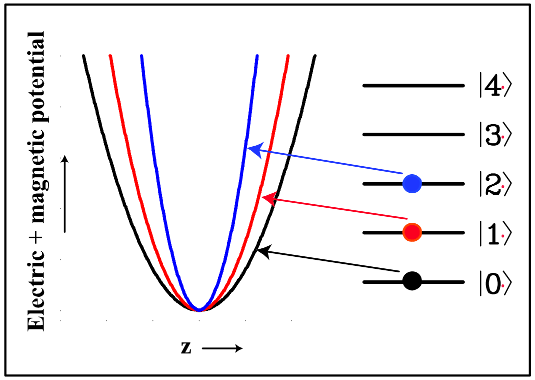


$$B_z = B_0 + B_2 z^2$$

$$U = \tfrac{1}{2} k_E z^2 + \mu_{s+c} B_2 z^2$$

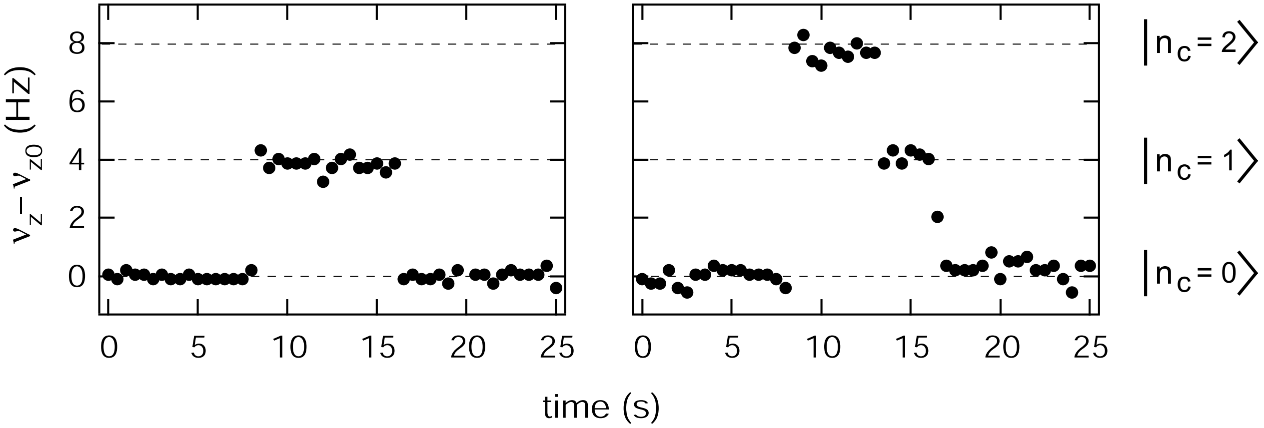
\+ Magnetic transitions are detected by axial frequency shifts.

\- A finite axial temperature smears the magnetic field.

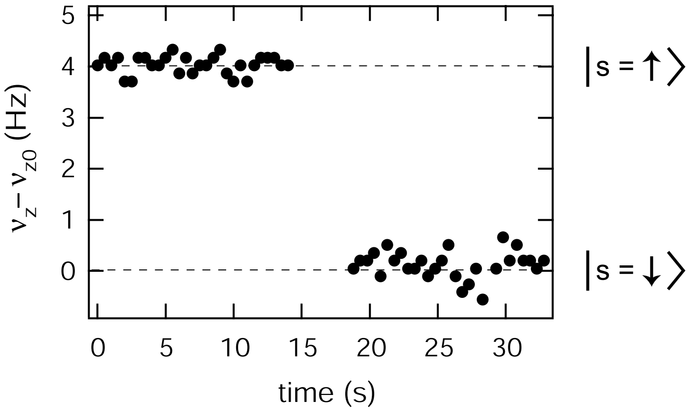


Quantum Jumps

Cyclotron Transitions:



Spin Flip

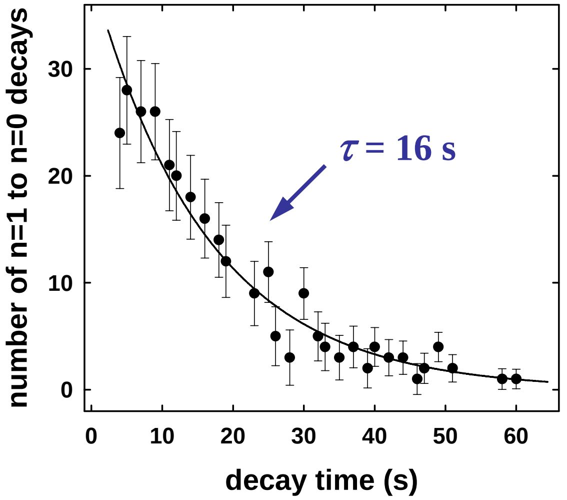


- The magnetic bottle allows us to make QND measurements of the cyclotron state.
- We can watch quantum jumps in real-time.

Inhibited Spontaneous Emission

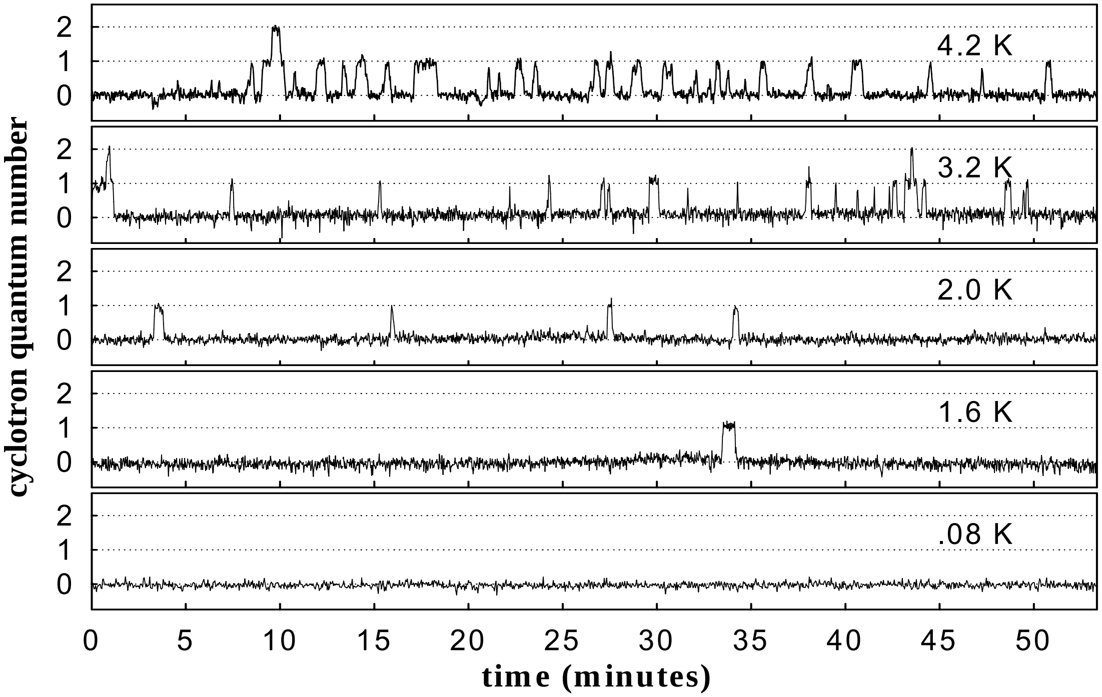
The free space cyclotron lifetime = 0.1 s.

We have achieved a 16 s lifetime.



Low Temperature

- •Running below 1 K leaves the cyclotron motion in the ground state all the time.
- •This allows for single quantum cyclotron spectroscopy, and all cyclotron transitions are driven, not thermal.
- •Low quantum numbers eliminate the relativistic broadening (1 ppb/quantum) of classical spectroscopy.



Thermal jumps are eliminated as the trap temperature is decreased from 4.2 K to 80 mK.

Cavity Modes

Hyperbolic Trap Problem

•Cavity modes shift ν_c

➢Systematic error in *g*-2

•Hyperbolic trap has low *Q* modes

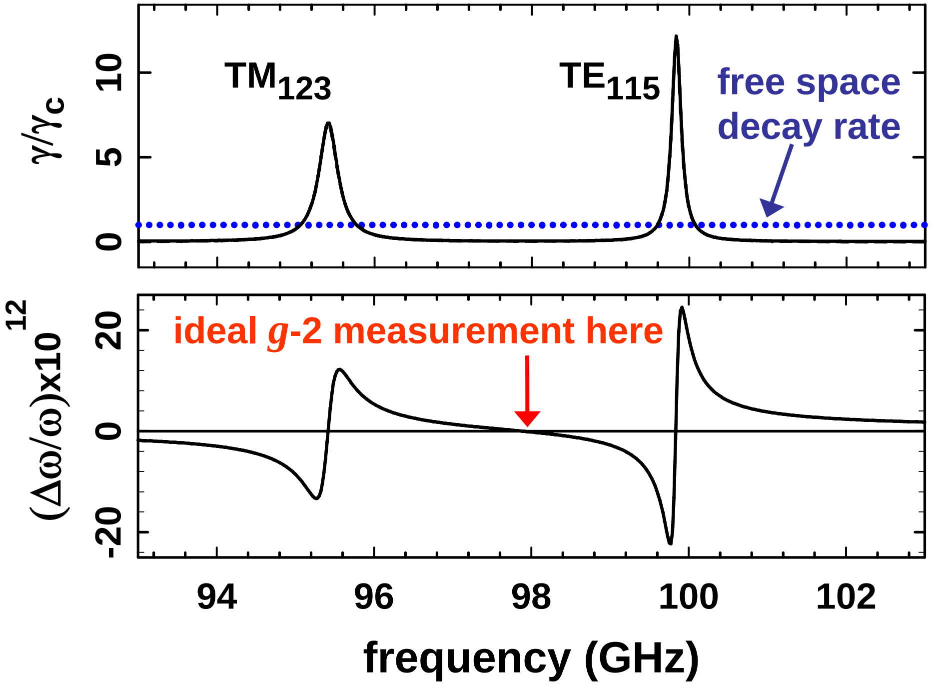
➢Electron and cavity interact for all values of the magnetic field

•Mode geometries are hard to calculate

➢Estimating error is difficult

•These problems led to a 3.8 ppb uncertainty in the 1987 University of Washington g-2 measurement

Cylindrical Trap Solution



•Cylindrical trap modes are much easier to calculate and identify.

•Operating at an appropriate spot between modes eliminates cavity-shift problems.

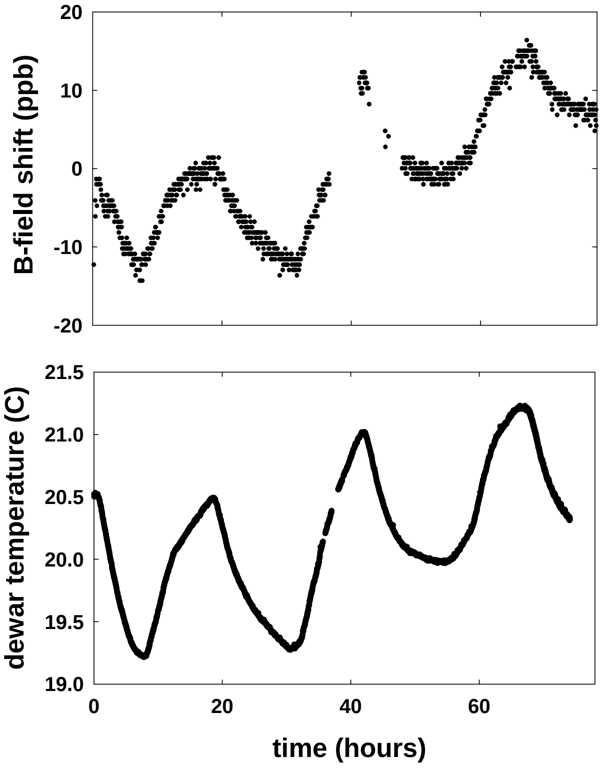
•The residual cavity shift is expected to contribute < 1 ppb to *g*-2.

Experimental Challenges

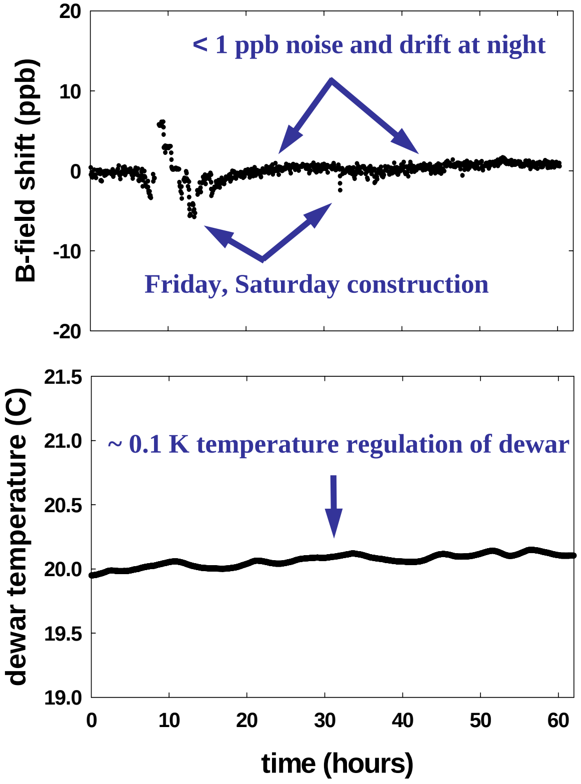
Magnetic Field Stability

Room temperature fluctuations affect the electric field seen by the electron because the magnet coils and the trap electrodes are mounted independently and can move relative to each other.

- •Magnet with two broken shim coils
- •No room temperature regulation



- •Well-shimmed magnet
- •Room temperature regulation to 0.1 K


< 1 ppb noise and drift at night

Friday, Saturday construction

~ 0.1 K temperature regulation of dewar

Other Challenges

- •Vibration
- ✓Nuclear paramagnetism of electrodes
- ✓Cryogen pressure regulation
- •Radiation leakage from 300 K causes cyclotron jumps

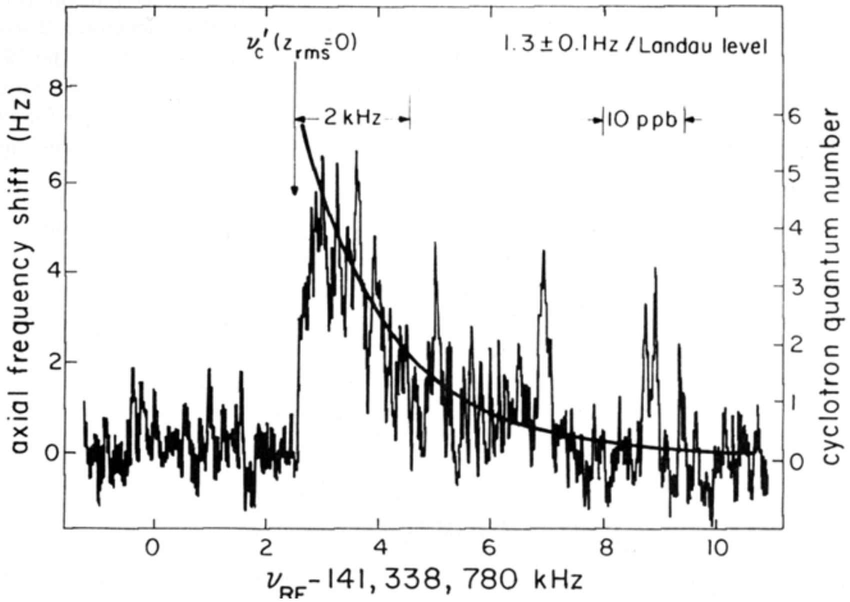
The Cyclotron Line

Procedure:

- With the electron in the N = 0 state, pulse the cyclotron drive (149 GHz).
- Look for excitations to N ≥ 1.
- Make a histogram of excitations versus frequency.

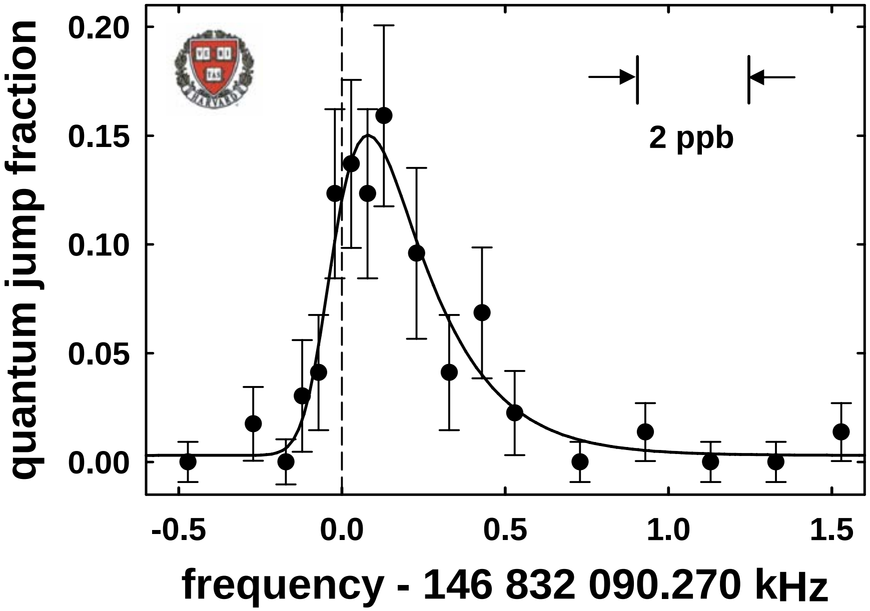
U. Wash. cyclotron line

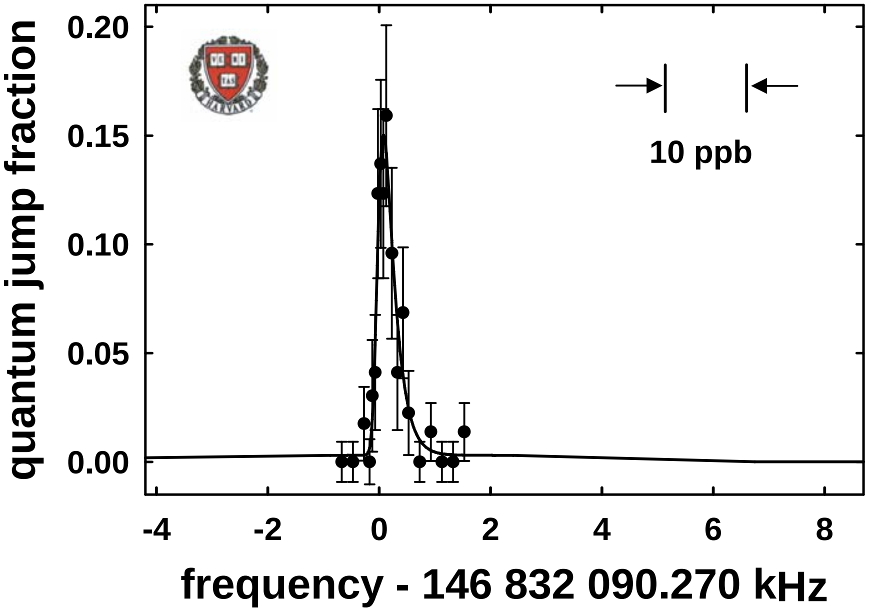
R.S.Van Dyck et al. *Phys. Rev. Lett.* 59, 26 (1987)



	U. Wash.	Harvard	Δ_H / Δ_{UW}
T_z (K)	6	0.6	0.1
υ_z (MHz)	60	200	0.09
B_2 (T/m²)	150	1500	10

Harvard cyclotron line





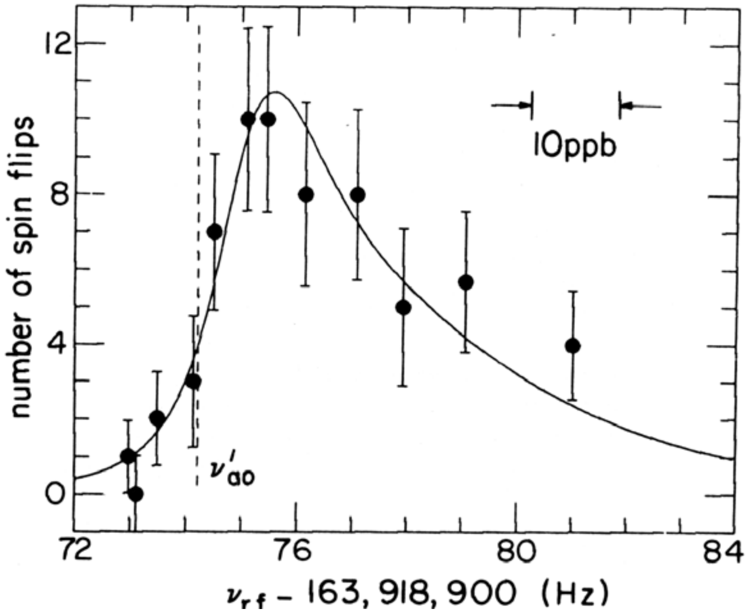
The Anomaly Line

Procedure:

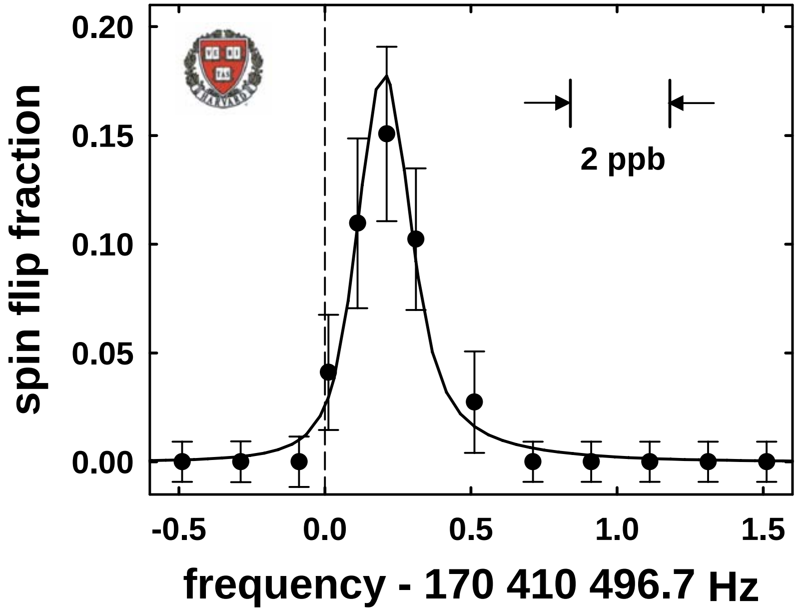
- With the electron in the |0,↑⟩ state, pulse the anomaly drive (172 MHz).
- Look for a transition to |1,↓⟩, which decays to |0,↓⟩.
- Make a histogram of spin flips versus frequency.
- In order to prepare for the next measurement, put the electron into the |0,↑⟩ state by applying the cyclotron and anomaly drives either simultaneously or sequentially.

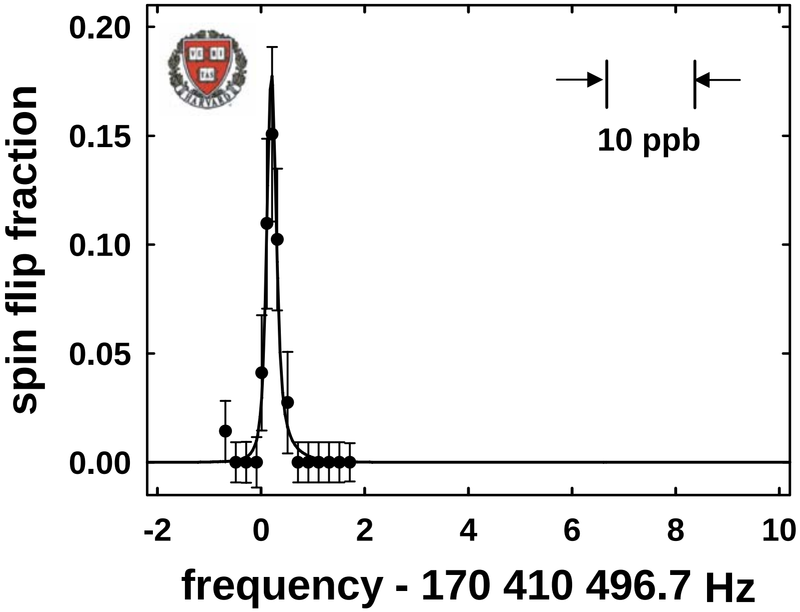
U. Wash. anomaly line

R.S.Van Dyck et al. *Phys. Rev. Lett.* 59, 26 (1987)



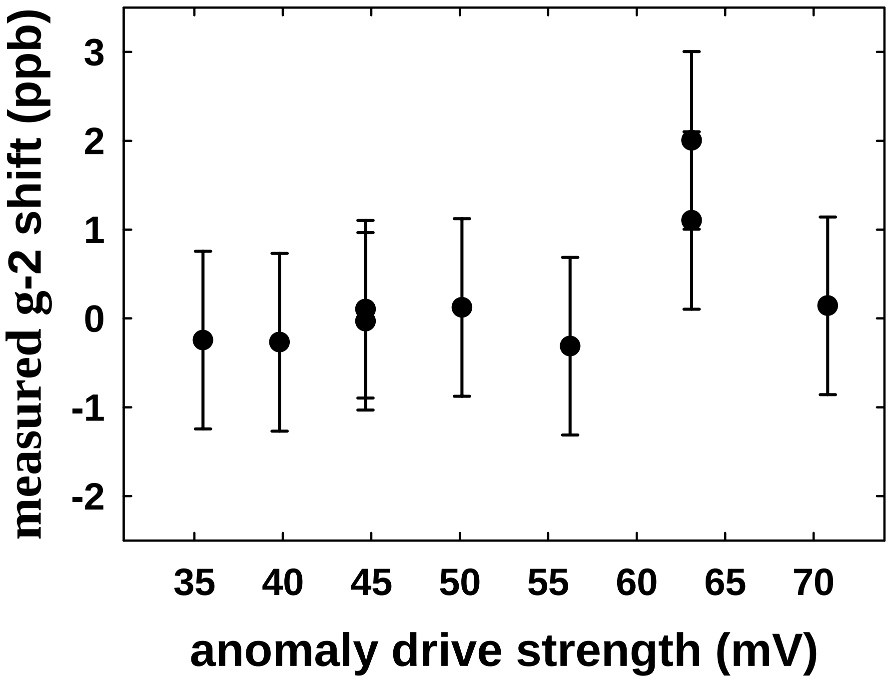
Harvard anomaly line



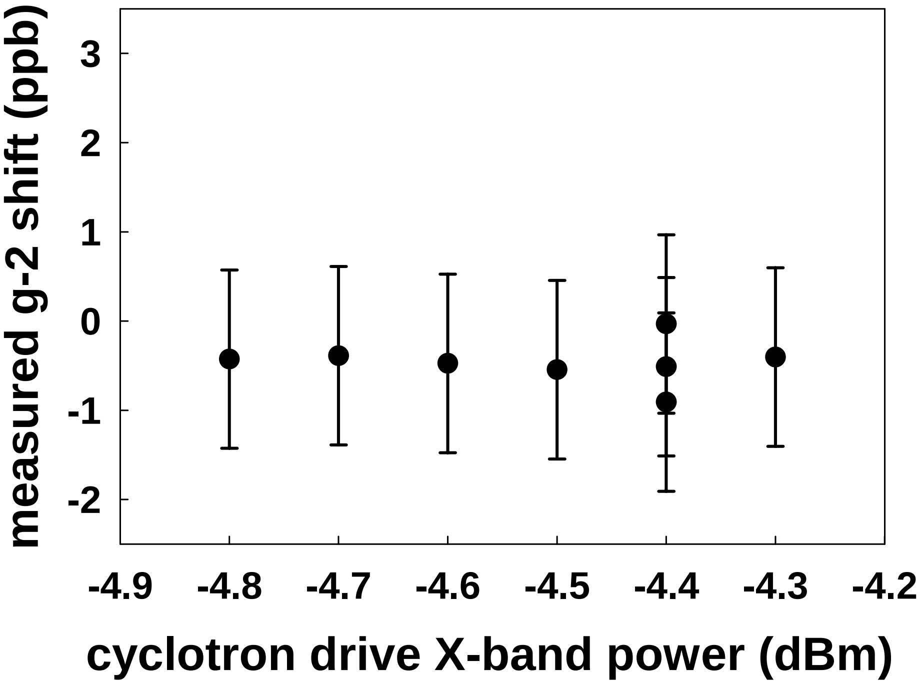


Power Systematics

Anomaly Drive Shift

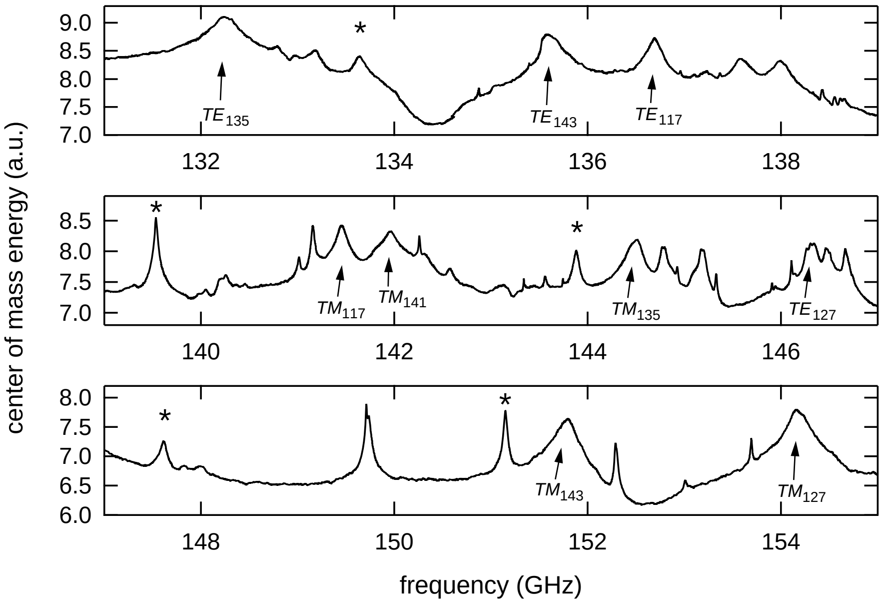


Cyclotron Drive Shift



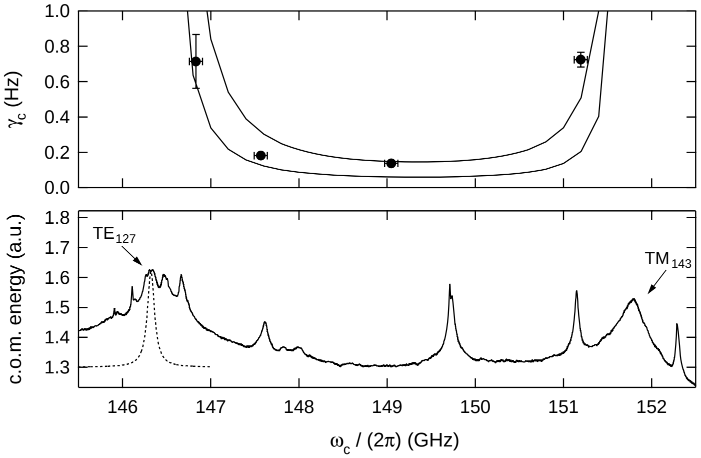
- *g*-2 is repeatable to better than 1 ppb.
- Power shifts look small.

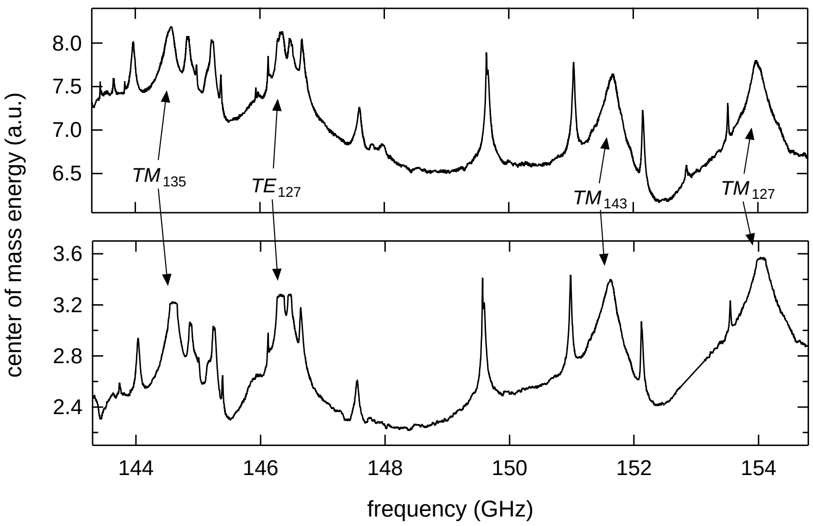
Mode Mapping



Measuring the cyclotron frequency on several features (*) allows us to calibrate our mode maps.

Our current region of interest:





Strongly-coupled modes saturate as the number of electrons is decreased from 3 x 10^4 to 1.6 x 10^4.

Parametric Excitation

Driving a cloud of electrons at twice the axial frequency allows us to probe the mode structure of the trap cavity. The modes cool the cyclotron motion of the electrons. The other degrees of freedom are coupled to the cyclotron motion through collisions, so they too are cooled, and the cloud's motion synchronizes.